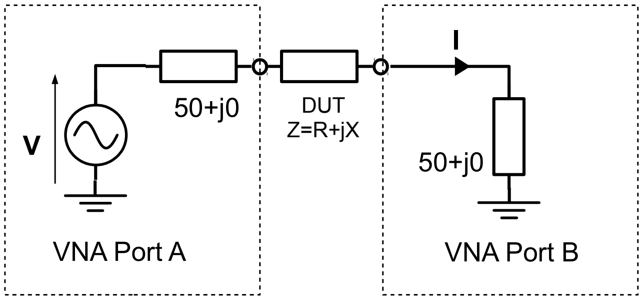
Derivation of R and X from S_{21} Magnitude and Phase for a device placed in series between the two ports of a VNA



Calibration

During calibration we replace the device under test (DUT) with a short circuit. The reference current into Port B then becomes:

$I_{cal} = V / 100$

Measurement

Let the DUT complex impedance be Z=R+jX, and let the total resistance in the VNA "series "loop" be r=R+100

Then magnitude of the current into Port B becomes:

$I = V / SQRT(r^2 + X^2)$

The change in current magnitude between calibration and measurement results in:

$|S_{21}| = 20.Log (I / I_{cal}) = 20.Log [100 / SQRT(r^2 + X^2)]$ .... (a)

and

S_{21} Phase = $\theta = -\tan^{-1} (X / r)$ .... (b) [Positive X causes lagging S21 phase]

Re-arranging (b):

$X = -r.\tan (\theta)$

Substituting this value of X in equation (a) gives:

$|S_{21}| = 20.Log [100 / SQRT(r^2 + r^2.\tan^2 (\theta))] = 20.Log [100 / r.SQRT(1 + \tan^2(\theta))]$

Re-arranging, and using the trig identity $1/SQRT(1 + \tan^2(\theta)) = \cos(\theta)$ gives:

$10^{(|S21| /20)} = 100.\cos(\theta) / r$

So:

$r = 100.\cos(\theta).10^{(-|S21| /20)}$

Then

DUT $R = r - 100 = (100.\cos(\theta).10^{(-|S21| /20)}) - 100$

And finally:

DUT $X = -r.\tan (\theta) = -(R+100).\tan (\theta)$