RECEPTION OF EXTENDED INCOHERENT SOURCES

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ABSTRACT - Moonbouncers often use solar and lunar noise to estimate the performance of their receiving systems. However, when the antenna beamwidth is comparable to the angular size of a noise source, the relation between gain (and hence G/T) and received noise level is not linear. Based on some simplifying assumptions, I try in this paper to calculate the expected level of received noise. This result is later applied also to the case of a reflected signal, as an attempt to investigate the usefullnes of big dishes for 10 GHz EME. (see the discussion in DUBUS 2/94)

1. List of variables

θ,ϕ	angles of the spherical coordinate system, θ is counted from the z axis.
$F(\theta,\phi)$.........	directional pattern of the antena, normalised so that maximaum $F(0,0)=1$
$T(\theta,\phi)$	angular distribution of noise temperature (brightness) of the source
Ω	solid angle
Ω_a..............	solid angle subtended by the binary radiation pattern of an equivalent antenna (an antenna with the same directivity, but whose pattern takes only the values 0 and 1)
Ω_l	solid angle subtended by the Moon (0.00006 steradian)
θ_{3dB}..........	3 dB beamwidth of the antenna
D................	directivity of antenna, to keep it simple, it is considered here to be equal to the gain
θ_l	half the angle subtended by the Moon (0.25° = 0.00436 radian)
T_{al}	part of the antenna noise temperature, that is due to the noise received from the Moon

2. Introduction

In June, I managed to get and old Andrew 10 ft dish, which I wanted to use for 10 GHz EME. It was quite bumped, so I wanted to find out if it can still work at 10 GHz. To estimate it's gain, I decided to compare it to a 0.9m dish, that was considered to be of a perfect shape. I did two measurements, first using the Astra TV satellite as a signal source, and then with the Sun. Astra was indeed a good 10 dB stronger on the big dish, but the Sun gave only 7 dB more noise. The only explanation, besides faulty instrumentation, was that the Sun was not a good point source any more. A look in "the Bible" (Kraus: Radio Astronomy) showed that the radio Sun is even bigger (abt 20% on 10 GHz) than the visual disk (0.5°), and that it's brightest around the rim, to get things worse. So, for further measurements, I sticked to Astra. With a 3m dish its signals are a good 30 dB above noise, so one can see the first sidelobes, and more important, the nulls between the main and side lobes. These nulls are very sensitive indicators of phase errors, be they from bad focusing or dish deformation. They should be as deep as posible, at least 30 dB on both sides, on a properly focused healthy dish. With an extended source like the Sun, you cannot see these nulls. But since the final test of sensitivity was the reception of lunar noise, I still needed to calculate in advance the expected noise level. As the experiment with the Sun showed, I couldn't just plug in the point source equations. So I tried this way:

3. Some aproximations of the expected noise level

In our case, coherent reception of an incoherent source, the noise level at the antenna terminals, expressed as the equivalent temperature is:

$$T_a = \frac{\int_{4\pi} T(\theta,\phi)\cdot\left|F(\theta,\phi)\right|^2 d\Omega}{\int_{4\pi}\left|F(\theta,\phi)\right|^2 d\Omega}$$

we can simplify this to:

$$T_a = \frac{A_{ef}}{\lambda^2}\cdot\int_{4\pi} T(\theta,\phi)\cdot\left|F(\theta,\phi)\right|^2 d\Omega$$

since

$$\frac{A_{ef}}{\lambda^2} = \frac{D}{4\pi} = \frac{1}{\Omega_A} \quad \text{and} \quad \Omega_A = \int_{4\pi}\left|F(\theta,\phi)\right|^2 d\Omega$$

In the case of the Moon, if we say that the Moon radiates according to the Lambert's law, then the whole apparent disk is of the same brightness and we can take $T(\theta,\phi)=T_l \approx 200K$, for $\theta<\theta_l$, and $T(\theta,\phi)=0$ elsewhere. Now it is easy to calculate the above integral in two extreme cases: (fig 1.)

1. when the angular size of the source is very small compared to the beamwidth of the antenna, we can take $F(\theta,\phi)=1$ and hence

$$T_{al}=T_l*\Omega_l/\Omega_a = T_l*\Omega_l*A_{ef}/\lambda^2. \qquad (1)$$

This means that the received noise power from the moon increases linear with the gain of the antenna. For 10 GHz and 50% efficiency, the dependency of lunar noise on dish diameter is:

$$T_{al} = 5.6 * d^2 , \qquad \text{d in meters} \qquad (2)$$

This is shown in the region "A" in the left part of fig 2.

2. when the angular size of the source covers most of the antenna pattern, then

$$\int_{4\pi}\left|F(\theta,\phi)\right|^2 d\Omega \approx \int_{\Omega_l}\left|F(\theta,\phi)\right|^2 d\Omega$$

and $T_{al} \approx T_l$, that means that further increase in size and gain gives us no more power. Another explanation for this can be taken from geometric optics: the parabolic mirror is an imaging device - the size of the image depends on the focal lenght, and its brightness on f/d ratio. When we increase the size of the mirror, and hence the focal length, the image of the moon also grows, until it is bigger than the mouth of the feedhorn. So on a big dish, the feed sees only a part of the moon, and cannot collect all the power that the dish collected.

But what does "most of the pattern" mean? I have studied a measured pattern of a 1.8m antenna on 1.7 GHz. A numerical integration showed that only about 65-70% of the power goes into the main lobe. That increases to about 80% if we include first and second side lobes. Therefore, the aproximation should be:

$$T_{al} = 0.75*T_l \approx 150 \text{ K} \qquad (3)$$

This is represented by the horizontal line (region "C") in the right part of fig 2.
1.8 m at 1.7 GHz is small in wavelenghts compared to 10 GHz EME antennas, but it was all I could get till now. When i'll get pattern measurements of an electrically bigger antenna, i'll repeat the calculations, but I don't expect much diference.

However, on 10 GHz, with dishes of about 2 to 6 m, we are just in the middle between the above cases! (region "B" of fig. 2.) That means, we must calculate the integral. The best would be to numerically integrate with the measured pattern, but that means you have to measure your pattern! The theoretical expression for the pattern of a round dish with tapered illumination is quite complicated, involving Bessel functions. Although it wouldn't be too hard to numerically integrate that, it is questionable how much precision one would gain compared to a simpler aproximation, considering that the actual pattern is distorted from its theoretical shape by shadow and diffraction effects, not to mention bumped dishes, HI. The simplest aproximation is a parabola, $|F(\theta,\phi)|^2 = 1-a*\theta^2$. If we match the 3 dB points, $a=2/\theta_{3dB}^2$. This aproximation is valid until the zeros of the parabola (at $\theta_0=\theta_{3dB}/\sqrt{2}$) hit the edge of the moon, and that happens when the diameter of the dish exceeds 6m. If we go further, we get a negative contribution, and that is certainly wrong. If we calculate the gain of an antenna with such a pattern, from zero to zero, we get approx 3dB more gain than a real dish with the same 3dB beamwidth. As stated above, if we integrate only the main lobe of a real dish, we get 2 dB too much gain. That would mean that this aproximation is about 1 dB or 20% low, when we integrate to zero point.
For integration, $d\Omega=d\phi d\theta \sin\theta$. Since we suppose rotational symmetry, there is no dependence on ϕ, and integration by ϕ only gives the factor 2π. With the small angles involved, we can say $\sin\theta = \theta$. Error is less than 1% for $\theta>22°$.

$$T_{al} = T_l 2\pi \cdot \frac{A_{ef}}{\lambda^2} \cdot \int_0^{\theta_l} \left(1 - \frac{2\theta^2}{\theta_{3dB}^2}\right)\theta \cdot d\theta = T_l \pi \cdot \frac{A_{ef}}{\lambda^2} \cdot \left(\theta_l^2 - \frac{\theta_l^4}{\theta_{3dB}^2}\right)$$

For 10 GHz and 50% efficiency, the moon noise contribution for a dish d meters across is:

$$T_{al} = d^2 (5.96 - 0.094 d^2) \qquad (4)$$

This is the curve in the middle of fig 2. This figure summarises the results of above calculations. The dashed curve is a guess of what happens above the validity range of the parabolic aproximation, but I think it must be quite close to the real thing. In practice, it is possible to achieve up to 65% overall efficiency with scalar feeds, so the curves in fig 2. could be a little higher. But to get those numbers, it will be necesarry to dig into measured patterns of real world high-efficiency big dishes. I hope i'll find time and energy in the near future to do that.

4. Reception of a coherent signal reflected off the Moon.

I wasn't able to find any reports on radar research of the moon - it was mainly done 30 -40 years ago - so the following is only a mind exercise, based on some more or less plausible assumptions!
Since the surface roughness of the Moon is large compared to 30mm wavelength, it is reasonable to assume that the reflected signal is random phase. That means that the reflected signal loses most of its spatial coherence, and the equations described above, for the reception of noise can be applied. That is because the partial signals reflected off different parts of the lunar surface arrive with uncorellated phase, and hence add in power, not in voltage.

Now, let's make another assumption, that is not very realistic, but it will help to explain some ideas. Later we will apply these ideas to a more realistic picture.
This assumption is, that the moon is also equally bright over the apparent disk in *reflected* radio waves.
If somebody illuminates the moon with a small antenna with a broad pattern, the lunar disk will be of constant brightness. For some other station, the received signal power will vary with dish diameter in a similar way as the thermal noise, as shown in fig 2. That means, that beyond a certain diameter (about 5 or 6m in this case) there will be no increase in the received signal!
Let's increase the size of the transmitting dish! So long as the beamwidth of the tx antenna is larger than the moon, the illumination of the moon stays even, but the brightness increases linearly with gain. The receiving station will see a corresponding increase in received level, but it still cannot get any reward from using dishes bigger than 5 or 6 m.
This situation doesn't change until the tx beamwidth becomes smaller than the Moon diameter. Now the brightness still grows linearly with gain, but the "spotlight" on the Moon also becomes smaller! As equation (1) tells, now it pays for the receiving station to increase the size of his dish! But only until its beamwidth matches the size of the "spotlight" on the moon. Beyond that, we again have the situation when the image of the source is too big for the feed and increasing the dish size has no sense.
Because of reciprocity, the same is valid in the other direction: if the receiving station has a constant dish size (say, just to cover the Moon), increasing the tx dish beyond that brings no benefit **-** the "spotlight" is brighter, but smaller, and as eqn (1) tells, the ratio of solid angles just compensates the increased brightness.
This can be summarised in three simple rules:

1. As long as the antenna beamwidths are wider than the moon, it pays to increase dish size on both sides. This is the usual situation on lower bands.

2. **When one of the stations has beamwidth less than the angular size of the moon, it is the SMALLER of the both dishes that determines the path loss!**

3. From 2. follows that with beamwidths narrower than the Moon, the optimum is when both sides have dishes of the same size. In this case they can increase the size of their dishes without limits and still gain on the signal. This naturally aplies to the own echo.

The criterion used here was that the Moon covers the whole main lobe. If we take the 3 dB beamwidth, the critical size of the dish decreases to some 4.5 m.

As told in the beginning, these conclusions were made on a unrealistic assumption about the Moon reflectivity. A more realistic assumption is, that the Moon reradiates the infalling power in a Lambertian way. In that case, the brightness of the Moon is dependent only on illumination. If we suppose that the transmitting pattern is constant over the angle subtended by the moon, the illumination is dependent only on the angle. For a round Moon, the brightness varies aproximately as

$$B(\theta) = c \cdot \sqrt{1 - \left(\frac{\theta}{\theta_l}\right)^2}$$

With this brightness distribution, half the power is reflected by the inner 61% (diameter wise) of the lunar disk. What does such an asumption change on the above three rules? First, since the reflection decreases more gradually toward the edge, the transition between the "linear" and "saturated" regions, where the increase in dish size still brings more signal, but not as much as could be expected from the gain increase, is less abrupt. Second, the above rules are aproximately valid for a 40% reduced beamwidth, and the "threshold dish size" is somewhere about 10m. (or about 6 m if we take the 3dB beamwidth)

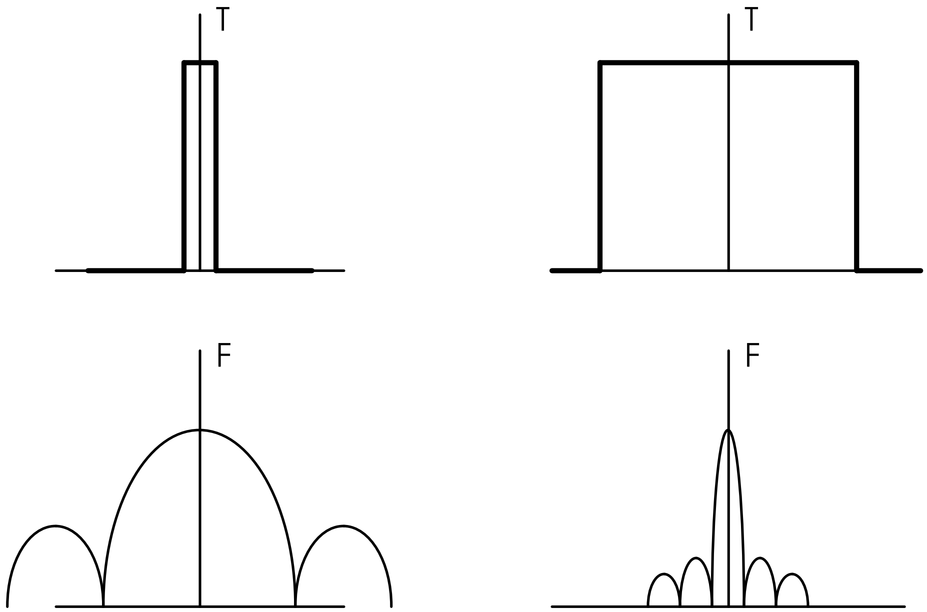
Another way to explain this "saturation effect" would be the van Cittert-Zernike theorem of spatial coherence. From it the distance over which the field of a distant extended random phase source is coherent can be calculated. In general, at a fixed distance a smaller source gives a larger area of coherence. That means, that for a large receiving aperture the apparent source at the Moon should be made small enough that its radiation is coherent across the rx aperture. This can be done by illuminating only a part of the Moon - hence a large rx dish also needs a large tx dish to be effective.

On the other hand, a big dish on only one side might still bring another benefit: the reduction in differential doppler, that smears the signal power across the frequency band, known as libration fading. When illuminating (or receiving from) only a samll patch in the middle of the apparent lunar disk, the range of different radial velocities gets smaller, and that means a narrower spectrum on receive.

5. Conclusion

The following conclusions can be made:

- for the measurements on dishes larger than 50 to 100 λ, it is better to stick to TV sattelites that are real point sources of plane waves.
- when estimating the G/T with lunar noise, caution is required in the transition region. For dishes bigger than 5 or 6 m, only the system temperature can be estimated this way!
- extremely big dishes bring no more signal power if the other station's dish isn't of comparable size, however they might reduce libration fading.



Top row represents sources, bottom row antenna directional patterns.

Left (wide beam, narrow source approximation):

$$\frac{1}{\Omega_a}\int_{4\pi}\left|F(\theta,\phi)\right|^2 T(\theta,\phi)\,d\Omega \approx \frac{T_l}{\Omega_a}\int_0^{\theta_l} d\Omega = T_l\frac{\Omega_l}{\Omega_a}$$

Right (narrow beam, wide source approximation):

$$\frac{1}{\Omega_a}\int_{4\pi}\left|F(\theta,\phi)\right|^2 T(\theta,\phi)\,d\Omega \approx \frac{T_l}{\Omega_a}\int_{\Omega}\left|F(\theta,\phi)\right|^2 d\Omega \approx T_l\frac{\Omega_a}{\Omega_a} = T_l$$

Fig 1.

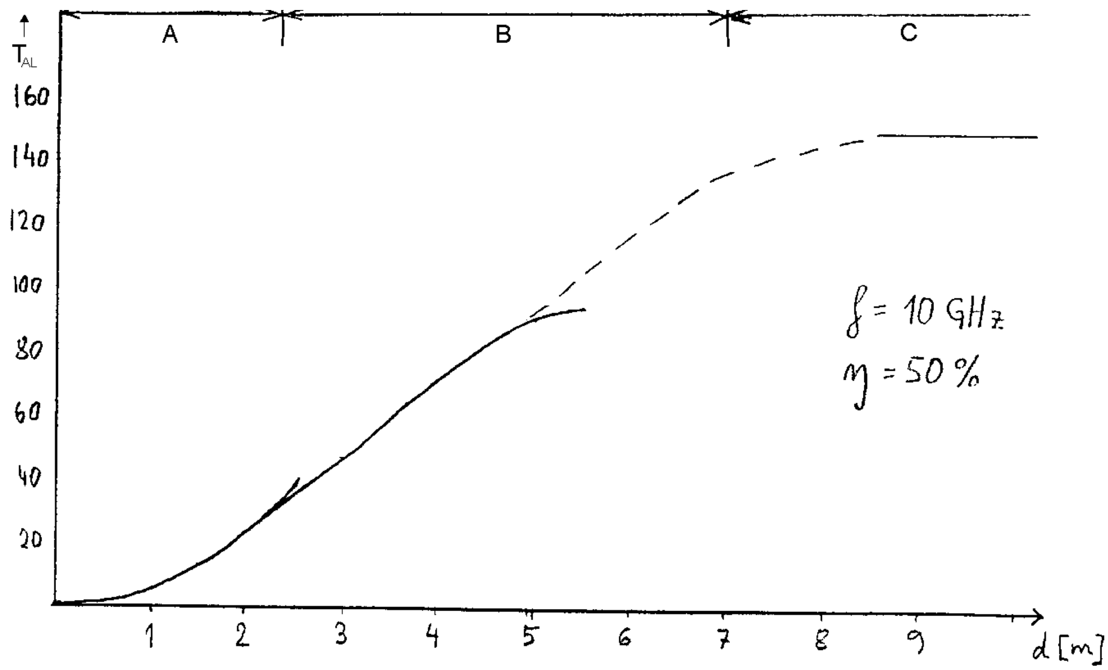


Fig. 2: Lunar contribution to antenna noise temperature as a function of dish size