The Philosophy and Formalization of the Informational

ANTON P. ŽELEZNIKAR
Volaričeva ul. 8, Ljubljana, SI–1111; E-mail: s51em@hamradio.si

Abstract. This paper deals with basic and advanced principles and consequences of the informational in a philosophical, formalistic (symbolic), schematic, and graphical way. The central notion is informational entity as the operand and its interaction (operational property) with other entities and itself. Informational externalism, internalism, metaphysicalism, and phenomenalism are principles constituting the informational conceptualism together with informational emergence and circularity of entities (operands). Formulas and formula systems are expressions for describing and interpreting situations and attitudes existing physically and phenomenally. Informational decomposition is provided for generating formulas and formula systems in the given informational environment. At last, informon is conceptualized as a conscious informational component of a conscious system. A new math-like formalism is introduced for the expression of simple and complex informational entities together with the new self-explaining symbolism.

Key words: complexity, conscious, decomposition, emergence, entity, externalism, formalism, formula, formula system, graph, informational, informon, internalism, metaphysicalism, operand, operator, phenomenalism, philosophy, rotation, shell, scheme, solution

1. Introduction

The 20-th century did not favor an explicit philosophy dedicated to the phenomenon of information or, more precise, informational entity. At the end of the former century the term Information Society became popular. It is evident that in several decades conscious robots (Moravec, 1999) and spiritual machines based on complex computational machinery and its exponential development (Kurzweil, 1999), with complex archives of knowledge and experience, will come in the everyday use (Knuth, 2002). On the average, the conscious human of information society (philosopher, computer scientist, 'informatician', citizen) begins to believe that *things, beings, events, processes, situations, attitudes — everything — not only 'are' in the sense of phenomenology, but inform in the postmodern sense of meaning.* The verb 'to be' is and can be replaced by the verb 'to inform'. In a live individual conscious system cognitive, emotional, behavioral, attentional, and other informational entities inform, that is, come to the conscious level. The informational pervades the physical and the phenomenal, can be observed, measured, and thought. The philosophy of the informational is offered to the human mind as a new discipline and field of research, as a new possibility of human physical and mental design and understanding.

This paper is, in fact, a recollection of author's endeavor concerning the philosophy and the new formalism of informational entities. In the last period of the research, the concept of the so-called informon was launched (Železnikar, 2002a; Železnikar, 2002b), understanding an informational component (entity) within a conscious system to be conscious by itself. This is possible because, for instance, an individual conscious system is sufficiently complex, and each of its informational components can take this kind of complexity for its own conscious property. Complexity within this texture means the complex informational interweavement of an extremely large number of components in the conscious system.

The program for a systematic development of the philosophy of the informational was traced in the second half of the 1980's by the publishing of a general plan of the principles of information (Železnikar, 1987; Železnikar, 1988a) and, more exhaustively and precise, by the book 'On the Way to Information' (Železnikar, 1990b). At that time, also the basic symbolism and formalistic expression was conceptualized, dealing with informational logic (Železnikar, 1988b; Železnikar, 1988c; Železnikar, 1989a; Železnikar, 1989b) and informational algebra (Železnikar, 1990a). Later on, a study of informational topology showed the possibilities and reasonableness of a mathematical topological view of the informational (Železnikar, 1998). A long-term study of the informationally conceptualized artificial consciousness is available (Železnikar, 2002c). In this view, the philosophy of the informational in this paper is only a recapitulation of some concepts occurring in the texture of philosophical, formalistic, and implementation possibilities.

2. The list of concepts and symbols

The following list is an overview and reminder of introduced concepts, their names and symbolic representation discussed in this paper. In this way, the initial image of theoretic essentiality pertaining to the field of the informational can be grasped in a skeleton way, including the systematic order of concept development. There is:

(1) Operands, α, β, $\ldots$, $\mathfrak{a}$, $\mathfrak{b}$, $\ldots$, $\mathfrak{A}$, $\mathfrak{B}$, $\ldots$, $\mathcal{A}$, $\mathcal{B}$, $\ldots$,with subscripts and superscripts.

(2) Binary operators (relators, quantors), $\models$, $\rightleftharpoons$, $\in$, $\prec$, $\forall$, and others, with subscripts and superscripts.

(3) Transition, primitive formula, $\alpha \models \beta$, in primitive formula system.

(4) Parenthesis pairs $(,)$, $[,]$, $\{,\}$, $\lfloor,\rfloor$, $\lceil,\rceil$, in explicitly and implicitly expressed formulas, formula systems, sets, formulas and systems of concern, etc.

(5) Formula, well-formed, $\varphi \rightleftharpoons \varphi\lfloor\alpha_1, \alpha_2, \ldots, \alpha_{n_\varphi}\rfloor$; circular, $\varphi^{\circlearrowleft}$; in more detail, $\varphi_{\triangleright}^{\triangledown}$, where $\triangledown \in \{\lambda, \circlearrowleft\}$ and $\triangleright \in \{\rightarrow, \leftarrow, \rightleftarrows, (\rightarrow, \leftarrow)\}$.
(6) Formula system, a set of formulas separated by semicolons; uniform, $\varphi^{\|}$, $\varphi^{\circlearrowleft\|}$, $\varphi_{\triangleright}^{\triangledown\|}$; mixed, Φ; primitive formula system, $\varphi^{\|'}$, $\varphi^{\circlearrowleft\|'}$, $\varphi_{\triangleright}^{\triangledown\|'}$, Φ'; system name, $\varphi^{\|^*}$, $\varphi^{\circlearrowleft\|^*}$, $\varphi_{\triangleright}^{\triangledown\|^*}$, Φ^*.
(7) Formula scheme, $\mathfrak{S}\lceil\varphi\rceil$, $\mathfrak{S}\left\lceil\varphi^{\circlearrowleft}\right\rceil$, obtained from φ, $\varphi^{\circlearrowleft}$, by omitting parenthesis pairs; the original formula is melted away in its scheme.
(8) Schematizing means deleting parenthesis pairs '(' and ')' in formulas, or forming formula schemes out of a complex graph along its paths.
(9) Scheme of a formula system, $\mathfrak{S}\lceil\varphi^{\|}\rceil$, $\mathfrak{S}\left\lceil\varphi^{\circlearrowleft\|}\right\rceil$, $\mathfrak{S}\lceil\Phi\rceil$, a system of formula schemes.
(10) Operand rotation, $\mathfrak{R}$, in a circular formula scheme, $\mathfrak{R}\left\lceil\mathfrak{S}\left\lceil\varphi^{\circlearrowleft}\right\rceil\right\rceil$, then parenthesizing, $\mathfrak{P}\left\lceil\mathfrak{R}\left\lceil\mathfrak{S}\left\lceil\varphi^{\circlearrowleft}\right\rceil\right\rceil\right\rceil$, to obtain a new circular well-formed formula.
(11) Graph of a formula, $\mathfrak{G}\lceil\varphi\rceil$, $\mathfrak{G}\left\lceil\varphi^{\circlearrowleft}\right\rceil$; circular formula delivers a circular graph (loop).
(12) Graph of a formula system, $\mathfrak{G}\left\lceil\varphi^{\|}\right\rceil$, $\mathfrak{G}\left\lceil\varphi^{\circlearrowleft\|}\right\rceil$, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\triangledown\|}\right\rceil$, $\mathfrak{G}\lceil\Phi\rceil$; the original formula system is melted away in its graph; formally, system graph is exactly described by corresponding primitive formula system, $\varphi^{\|'}$, $\varphi^{\circlearrowleft\|'}$, $\varphi_{\triangleright}^{\triangledown\|'}$, Φ'; several system graphs can be fused in one graph, where their identity is melted away.
(13) Subgraph $\mathfrak{G}_1$ of graph $\mathfrak{G}$, $\mathfrak{G}_1 \prec \mathfrak{G}$, is surrounded by the subgraph envelope, $\mathfrak{e}_{\text{envelope}}$, being the subgraph's outmost loop, usually an initial informational decomposition, concerning a concrete operand.
(14) Solution $\mathfrak{s}_{\text{solution}}$ concerning operand α occurring in a complex (fused) uniform and mixed formula system, $\varphi_{\triangleright}^{\triangledown\|}$ and $\Phi\lceil\alpha\rceil$, respectively, is the complex meaning of α, expressed out of the system graph $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\triangledown\|}\right\rceil$ and $\mathfrak{G}\lceil\Phi\rceil$, respectively, as a uniform or mixed formula system, $\mathfrak{s}_{\text{solution}}\lceil\alpha\rceil \rightleftharpoons \mathfrak{P}\left\lceil\mathfrak{R}\left\lceil\mathfrak{S}\left\lceil\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\triangledown\|}\right\rceil\right\rceil\right\rceil\right\rceil$ and $\mathfrak{G}\lceil\Phi\rceil$ and $\mathfrak{s}_{\text{solution}}\lceil\alpha\rceil \rightleftharpoons \mathfrak{P}\lceil\mathfrak{R}\lceil\mathfrak{S}\lceil\mathfrak{G}\lceil\Phi\lceil\alpha\rceil\rceil\rceil\rceil\rceil$, respectively.
(15) Decomposition concerning something informational α is an intentional generation of formulas, formula systems, schemes, graphs, with the aim to acquire α's meaning as general, metaphysicalistic, informonic decomposition, etc., $\Delta\lceil\alpha\rceil$, $\mathfrak{M}\lceil\alpha\rceil$, $\Delta\lceil\alpha\rceil$, $\mathfrak{I}\lceil\alpha\rceil$, etc.
(16) Informon $\underline{\boldsymbol{\alpha}} \rightleftharpoons \left(\alpha; \underline{\boldsymbol{\mathcal{I}_{\alpha}}}, \underline{\boldsymbol{\mathcal{C}_{\alpha}}}, \underline{\boldsymbol{\mathcal{E}_{\alpha}}}\right)$, complexly and consciously organized operand, named α, with informonic components of intentional informing, counter-informing, and informational embedding.

This list of terms shows the course of the discussion in this paper.

Usually, the philosophy and formalization of the informational seems to be premature (Floridi, 2002) for researchers in philosophy, computer science, AI and mathematics. Namely, it has its own

- topics (facts, observations, problems, phenomena, emergent entities, circular causality, complexity, etc.),
- methodology (formalism of presentation, parenthesizing, decomposition, schematizing, operand rotation, graphs, etc.), and

- theory (principles, consequences, interpretive proofs, explanation, formalization (solution) of meaning, etc.).

For instance, the field of the informational is hardly understood as the space of everything that informs and is being informed; informational entity is not grasped similarly to the entity in phenomenology; and the new formal apparatus of the informational is disputed as inadequate on the mathematical level because of its novel logic, symbolism, and formal expression.

Concepts in this paper are not meant and envisioned to be definite. They are paused on the halfway to the possible or desired completeness because the full background of the informational might only be uncertainly anticipated. Several kinds of uncertainty comes fore as a consequence of formal representation. The most obvious example of this sort is a precise and sufficiently effective definition of the informational concern in the form $\alpha\lceil\beta\rceil$ which reads 'α of β', 'α informationally concerns β', etc.

3. Information, the informational, and informational entity

Information is usually a static concept in the sense of data and meaning. As such, a written or spoken sentence comes fore as information, a collection of interdependent (connected) data entities. Here, a special meaning of the term 'information' is important that does not represent a possible dynamic nature of information in itself. In this sense 'information' is instruction, intelligence, news, facts, scientific knowledge, and communication of such entities. This sort of meaning of 'information' is especially grasped in English. In other languages, the term information can have a much more general and other meaning, including that of an informationally dynamic phenomenon.

What could be the difference between the described meaning of information and that of the informational? The informational is everything that informs, that informationally impacts similar and other sorts of objects. In the emerging philosophy of the informational certain keywords play an essential role: spontaneity, emergence, and circularity of that which informs, also understood as informational spontaneism, emergentism, and circularism. Spontaneity means unforeseeability, unpredictability, indeterminism, chaos, randomness, an the like. Emergence means informational arising of entities in a spontaneous and circular way. Circularity means informational recurrence, iteration, repeating, returning, etc. However, let us formulate the principles of the informational more explicitly.

The informational is everything that can inform and can be informed, that is, can impact and can be impacted informationally. Things are informing, observers inform and are being informed by things, for instance. Physically, things impact and are impacted by gravitation, electromagnetism, quantum dynamics, etc. Informational entities (ideas, thoughts, experiences) within a conscious system inform and are informed, change other entities and are changed. The mind is an evident system of the informing of informational entities. As understood lexically, data informs, but cannot be informed (to be informed, data would lose its factuality, everlasting, informational stability). This should be valid also for facts and truth itself. In these cases, the informational dynamics of entities fails.

The concept of informational entity becomes evident since the phenomenological studies of Husserl and Heidegger. The term 'entity' is borrowed from phenomenology where its original meaning is 'Seiendes', that what 'is' (the being of something), that what 'is existent', that what 'informs meaningly'.

The term 'informational entity' concerns the domain of the informationally existent and the informationally organized. Informational entity emits and accepts to it specific (characteristic) information. In general, it informs and is being informed. It emerges, changes, and vanishes spontaneously and circularly.

Properties of informational entity are founded by three basic principles which rule its nature: 'informational externalism', 'informational internalism', and 'informational metaphysicalism'. Externalism means the possibility to be expressed informationally (output) while internalism means the introspection of that what arrives informationally to the entity (input). Further, informational entity behaves to itself informingly and informedly. It is the informer and observer to itself. This quality of informational entity is termed metaphysicalism. The connotation of the metaphysical to this quality of informational entity roots in the interior subjectivity of the entity understood as ontological, epistemological and cosmical unity concerning its meaning. In a conscious system, entities are organized in a metaphysicalistic way. Literally, this property can also be understood as being beyond the physical, e.g., being phenomenal.

Informational entity exerting externalism, internalism and metaphysicalism is termed 'informational phenomenalism' being a consequence of the introduced principles. The connotation of the phenomenal to this quality of informational entity roots in the interior and exterior performance of the entity as an informational phenomenon.

As it will be shown, symbolically-formalistically, informational entity is a whole (unit) captured by a system of formulas informationally linked by common operands. The operand linkage regularly extends to other informational entities as systems of formulas and make them meaningly connected on the level of formula systems.

Let us present an example. A headword or a nominal phrase within a sentence is informational entity. It represents a meaning within a natural language. It performs as an operand. In a sentence, operands are connected by operators (verbs, verbal phrases). Sentence is another informational entity which includes operands and operators and and approaches the concept of informational formula. Sentence has it specific meaning as a context of words. In a text, sentences are linked by common operands, forming another entity with its specific dynamic and complex meaning. Such an entity is formalized and termed abstractly the 'formula system'. To resume: entities are operands, formulas, and formula system, in this case.

However, informational entity is everything sensed and experienced as unity, e.g., image, music performance, cognition, emotions, behavior, discourse, understanding, that is, anything informational belonging to the real and phenomenal world. The informational supervenes on the physical and the phenomenal supervenes on the informational, in this order.

4. Why the philosophy and formalization of the informational

The philosophy of the informational is an informational theory of everything phenomenalizing informationally. It covers concepts, principles, axioms, consequences, conclusions, theorems, lemmas, proofs, and theoretical discourses concerning the informational. Why the philosophy of the informational is necessary nowadays when philosophers, theoreticians, and researchers recognize the existence of the so-called Information Theory and call for the establishing of the Theory of Information (Floridi, 2002)? The difference between the two should be in distinguishing a data theory and a dynamic concept of information. The philosophy of the informational moves further into the domain of mind and, especially, consciousness—the natural and the artificial.

Complexity and emergemce of entities are two essential principles of the informational. Informational complexity of informing entities is the sine qua non of conscious informational systems and their components (cognition, emotions). Emergence (arising) of informational entity might be best explained by individual conscious experience when the subject concentrates on thinking a certain object, its explanation, gaining and acquiring meaning through analysis, synthesis, association and other transcendetal, reflexive, emotional, cognitive and sensory informational behavior. In such a case, informational entity emerges in a possible complexity and organization, becomes informational perplexedness with other entities.

Qualities of informational emergence can be summarized by several principles. Informational emergence of entities happens spontaneously in serial, parallel and circular way. This means that informational entity as a formula emerges serially by extending its formula by additional operators and operands, that in a formula system additional formulas emerge, and that formulas are circularly closed and mutually linked by common operands. Circularity of informational entity means that operands inform in a circularly structured causal way.

For instance, natural language is a circular system in which words and higher structures are interpreted by words. In such a language, circularity of informing is direct (in a sentence) and transitive through the occurrence of common operands (headwords, nominal phrases) in sentences of the language.

For the informationalist, consciousness is an informational phenomenon. The only possibility today to design and implement the artificial consciousness is to make it informational—conceptually and technically. Also, studying, theorizing, researching, and experimenting in the domain of the live (natural) consciousness leads to the recognizing of the informational nature of consciousness. The most evident phenomena are cognition and emotions, as we experince them in our individual minds. There is no a counter-conceptual idea how to imagine the functioning of human mind otherwise, that is, by non-informational means and concepts (e.g., purely physically, biologically, non-linguistically). Consciousness remains on the way of the informational.

Formalization of the informational is realized by the innovative symbolic language called $\mathfrak{Z}$-language, using operands, operators, parenthesis pairs, formulas and formula systems together with the new methodology concerning complexity and emergence of formal entities. $\mathfrak{Z}$-language is provided for the most precise expression and design of complex and emergent informational situations reaching up to the level of conscious informational systems and conscious components. The letter $\mathfrak{Z}$ symbolizes the informing of the entity known in natural language as consciousness and being denoted by $\mathfrak{z}$ (instead to use the subscribed operand $\mathfrak{c}_{\text{consciousness}}$, for example). The $\mathfrak{Z}$-language is aimed for conceptualization and design of artificial conscious systems as well as for identification and study of human and animal consciousness, where cognitive, emotional, and other components inform as complex and emergent conscious entities. Within this context of expressional and descriptive possibilities, $\mathfrak{Z}$-language can contribute substantially to the study in language, information, philosophy, psychology, clinical psychiatry, innovative mathematics, and other information-dependent disciplines.

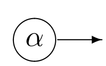
5. Basic principles of the informational

The theory of the informational, as a part of the philosophy of the informational, demonstrates a new formal symbolism as a mathematical possibility of expression of informational formulas and formula systems. Both are descriptions of informational situations behind which processes, events, and experiences of informing of entities can be imagined. In this view, formal informational descriptions are emergent by themselves and represent emergent informational entities. The dynamics roots in the nature of informational operands (entities) and informational operators mutually and differently depending from one situation to the other. Both operands and operators carry the potentiality of development called informational decomposition.

The basic informational principles or axioms deal with the expression of the most basic formulas or primitive transitions of the form $\alpha \models \beta$. A trivial expression or formula is the symbol of a certain operand denoted by α. The length of such an operand formula is $\ell_\alpha = 0$. What we need is an operator symbol $\models$ with the meaning *inform(s)* or *is (are) being informed by.* This operator is a joker for any concrete operator, being meaningly exactly determined by a subscript. Informational operators are consequently *binary* operators describing the types of informing between operands; their meaning depends on the concrete left and right operand. To put down the basic principles or formal axioms we need the second general operand, being a joker for something not determined yet at all and symbolized by $\square$ (a yet empty informational framework). A logical consideration supports the conclusion in case when we say that an entity informs (something), yielding formally, $\alpha \models$. If something informs there must exist something being informed. That is, on the right side of operator $\models$, the empty place is represented by the yet concretely unknown informational entity $\square$, formally, $\alpha \models \square$.

A similar situation happens if we say that entity α is being informed, formally, $\models \alpha$. The informing comes from another, yet concretely unknown source of informing, denoted by $\square$. This yields $\square \models \alpha$. The two basic principles of informing can now be formulated formally.

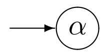
Principle 1 *Every informational entity, named α, informs. Formally,*

$$\alpha \models \square \quad \textit{or, graphically,}$$  .

This principle of an informational entity α to inform something $\square$ is called the informational externalism. □

Principle 2 *Every informational entity, named α, is being informed. For-*

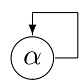
mally,

$$\Box \models \alpha \quad \textit{or, graphically,}$$  .

This principle of an informational entity α of being informed by something $\Box$ is called the informational internalism. □

We need a basic principle for making possible the inner development (emergence, decomposition) of an informational entity at the very beginning (a pure operator not further developed yet), when the decomposition will start from a trivial circular situation.

Principle 3 *Every informational entity, named α, informs itself and is being informed by itself. Formally,*

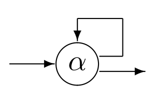
$$\alpha \models \alpha \quad \textit{or, graphically,}$$  .

This principle of an informational entity α to inform itself and to be informed by itself is called the informational metaphysicalism. □

Informational metaphysicalism explicates the circular nature of informational entity. This basic and apparently trivial circularity is significant for further development of the entity and, in the case of data, for its informational invariability.

Evidently, the three basic principles can be joined into the consequence constituting the phenomenalistic nature of informational entity.

Consequence 1 *According to Principles 1–3, every informational entity, named α, informs to something and itself and is being informed by something and itself. Formally, all these basic properties yield a formula system (the parenthesized form of the three primitive formulas separated by semicolons)*

$$\begin{pmatrix} \alpha \models \Box; \\ \Box \models \alpha; \\ \alpha \models \alpha \end{pmatrix} \quad \textit{or, graphically,}$$  .

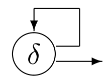
This principle of an informational entity α general informing is called the informational phenomenalism. *Informational entity informs to the exterior entities, is being informed by exterior entities to its interior, and informs and is informed within itself.* □

Proof 1 Every informational entity α informs to its externality (Principle 1), is being informed by its externality (Principle 2), and informs and is being informed by itself (Principle 3). QED. □

According to this basic consequence, every informational entity α informs in a phenomenalistic way, that is, within itself and its informational environment. The possible interior organization of α is a consequence of informational decomposition and will be discussed later. However, α informs circularly in itself and with its informational environment.

The difference between data δ and informational entity α can be clearly expressed. The data phenomenalism is

$$\begin{pmatrix} \delta \models [\,]; \\ [\,] \not\models \delta; \\ \delta \models \delta \end{pmatrix} \quad \text{or, graphically,}$$



This formal system directs to the following explanation. Data is not being informed, $[\,] \not\models \delta$, cannot be informed by exterior entities or cannot internalize exterior information. However, it has its own metaphysicalism, $\delta \models \delta$, in maintaining that what data represents exteriorly. This means that an inner decomposition (emergence, development) of data is not possible. Data is innerly stable, unchangeable, given as an absolutely stable informational entity. Thus, in the basic data system, formula $[\,] \not\models \delta$ could be simply omitted. On the other side, a kind of non-informing can represent an essential property of the entity in question. For instance, the input arrow in data graph can be marked by $\not\models$.

6. Advanced principles of the informational

The three basic principles and the basic consequence presented in the preceding section constitute together with logical operators and parenthesis pairs the informational disciplines called informational logic, informational algebra, and informational syntax or well-formedness. The last leads to the concept of informational formula.

Principle 4 *Informational formula φ is a well-formed sequence of operands $\alpha_1, \alpha_2, \ldots, \alpha_{n_\varphi}$, operators $\models$ and parenthesis pairs '(' and ')', denoted by the expression $\varphi\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\varphi} \rfloor$. The formula length is denoted by ℓ_φ, represents the number of operators in the formula, and depends on the formula type.* □

Usual formula types are denoted by $\varphi_{\rightarrow}, \varphi_{\leftarrow}, \varphi_{\rightleftarrows}, \varphi_{\rightarrow,\leftarrow}, \overset{\circlearrowleft}{\varphi}_{\rightarrow}, \overset{\circlearrowleft}{\varphi}_{\leftarrow}, \overset{\circlearrowleft}{\varphi}_{\rightleftarrows}, \overset{\circlearrowleft}{\varphi}_{\rightarrow,\leftarrow}$ (for details see (Železnikar, 2002c)). They denote serial ($\rightarrow$), reverse serial ($\leftarrow$), biserial ($\rightleftarrows$), split biserial ($\rightarrow, \leftarrow$), and circular ($\circlearrowleft$) formulas of these types, respectively. Within the formula length ℓ_φ, the number of possible differently parenthesized formulas is $\frac{1}{\ell_\varphi+1}\binom{2\ell_\varphi}{\ell_\varphi}$. The serial formulas out of these possibilities are from the lowest to the highest form of parenthesizing

$$\begin{array}{l} \varphi_{1\rightarrow}\lfloor \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n_\rightarrow -1}, \alpha_{n_\rightarrow}\rfloor \rightleftharpoons \\ \quad (\alpha_1 \models (\alpha_2 \models (\alpha_3 \models \ldots \models (\alpha_{n_\rightarrow -1} \models \alpha_{n_\rightarrow})\ldots)))\,;\ \ldots; \\ \varphi_{\frac{1}{n_\rightarrow}\binom{2(n_\rightarrow -1)}{n_\rightarrow -1}\rightarrow}\lfloor \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n_\rightarrow -1}, \alpha_{n_\rightarrow}\rfloor \rightleftharpoons \\ \quad ((\ldots((\alpha_1 \models \alpha_2) \models \alpha_3) \models \ldots \models \alpha_{n_\rightarrow -1}) \models \alpha_{n_\rightarrow})\,, \end{array}$$

where the serial formula length is $\ell_\rightarrow = n_\rightarrow - 1$. A concrete formula parenthesizing determines exactly the formula meaning. In general, different formula meanings are determined by possible formulas $\varphi_{i\rightarrow}\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\rightarrow}\rfloor$, where $1 \leq i \leq \frac{1}{n_\rightarrow}\binom{2(n_\rightarrow -1)}{n_\rightarrow -1}$. It is understood that operators in all these formulas are equally particularized (subscribed).

Formulas are abstract representatives of sentences, images, sound complexes, and other possible organization of informational structures. Irrespective of its complexity, every English sentence can be represented by an informational formula or a formula system (set of formulas) as precisely as necessary or more. The meaning of a sentence can be described by formulas as close as necessary and in more details by an additional interpretation using additional formulas. Some practice, rules, conventions, and a appropriate theory are needed in translating a natural language into the language of informational formulas, $\mathfrak{I}$-language, and vice versa. It is important that the meaning of an original sentence is not reduced or substantially modified.

Let's take the nominal phrase "that thin blue grammar book in plastic covers that I borrowed from our library." Here, 'book' is the headword with successive premodifiers 'grammar', 'blue', 'thin', and 'that'. We have an evident expression of informational concerning (parenthesis pairs '$\lceil$' and '$\rceil$') of the $\mathfrak{I}$-form

$$\begin{array}{l} (\mathfrak{b}_{\text{book}}\lceil \mathfrak{g}_{\text{grammar}}\lceil \mathfrak{b}_{\text{blue}}\lceil \mathfrak{t}_{\text{thin}}\lceil \mathfrak{t}_{\text{that}}\rceil\rceil\rceil\rceil \models_{\text{be\_in}} \mathfrak{c}_{\text{cover}}\lceil \mathfrak{p}_{\text{plastic}}\rceil) \rightleftharpoons \\ \quad (((((\mathfrak{b}_{\text{book}} \models_{\text{include}} \mathfrak{g}_{\text{grammar}}) \models_{\text{be}} \mathfrak{b}_{\text{blue}}) \models_{\text{be}} \mathfrak{t}_{\text{thin}}) \models_{\text{be}} \mathfrak{t}_{\text{that}}) \models_{\text{be\_in}} \\ \qquad (\mathfrak{c}_{\text{cover}} \models_{\text{be\_in}} \mathfrak{p}_{\text{plastic}})) \end{array}$$

Operator $\rightleftharpoons$ reads 'mean(s)', all operands and operators are subscribed (determined, particularized). On the right side of $\rightleftharpoons$ is the corresponding well-formed formula. The remained part of the phrase, "that I borrowed from our library," concerns the same headword, so 'that' in this part has the meaning of 'book'. Thus, the second formula, considering the postmodifiers of 'book', is

$$(\mathfrak{b}_{\text{book}} \,=\!\!|_{\text{borrow\_by}}\, \mathfrak{i}_{\text{I}}) \models_{\text{be\_from}} (\mathfrak{b}_{\text{book}} \models_{\text{be}} \mathfrak{o}_{\text{our}})$$

In the last formula, the alternative operator $\dashv$ to operator $\models$ is used which reads 'is_being_informed_by', in general ('is_being_borrowed_by', in this case). By the way, operators concerning tenses can use a special superscript marking the tense. E.g., α borrowed β yields the formula $\alpha \models^{\mathrm{past}}_{\mathrm{borrow}} \beta$. On the other side, operators can be subscribed by forms of verbs or verbal phrases as they appear in the original sentence.

Different informational formulas can have common operands. Such formulas can be reasonably fused into formula systems. Formula systems are arrays (sets) of formulas, separated by semicolons and enclosed into the parenthesis pairs, usually listed vertically for the sake of transparency. Systems can consist of uniform or mixed formulas. Uniform formula systems are denoted, adequately to formula denotation (see ahead), by $\varphi^{\parallel}_{\rightarrow}$, $\varphi^{\parallel}_{\leftarrow}$, $\varphi^{\parallel}_{\rightleftarrows}$, $\varphi^{\parallel}_{\rightarrow,\leftarrow}$, $\varphi^{\circlearrowleft\parallel}_{\rightarrow}$, $\varphi^{\circlearrowleft\parallel}_{\leftarrow}$, $\varphi^{\circlearrowleft\parallel}_{\rightleftarrows}$, $\varphi^{\circlearrowleft\parallel}_{\rightarrow,\leftarrow}$ whole mixed formula systems are denoted simply by Φ. Symbol $\parallel$ marks the parallel formula structure of the system.

Principle 5 *A uniform and mixed formula system, $\varphi^{\triangledown\parallel}_{\triangleright}$ and Φ, respectively, is a set of formulas $\varphi^{\triangledown}_{1\triangleright}, \varphi^{\triangledown}_{2\triangleright}, \ldots, \varphi^{\triangledown}_{n_{\varphi^{\triangledown\parallel}_{\triangleright}}\triangleright}$ and $\varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi}$, respectively, separated by semicolons and denoted by*

$$\varphi^{\triangledown\parallel}_{\triangleright} \rightleftharpoons \begin{pmatrix} \varphi^{\triangledown}_{1\triangleright}; \\ \varphi^{\triangledown}_{2\triangleright}; \\ \vdots \\ \varphi^{\triangledown}_{n_{\varphi^{\triangledown\parallel}_{\triangleright}}\triangleright} \end{pmatrix}, \quad \begin{array}{l} \textit{where } \triangleright \in \{\rightarrow, \leftarrow, \rightleftarrows, (\rightarrow, \leftarrow)\} \\ \textit{and } \triangledown \in \{\lambda, \circlearrowleft\}, \textit{ and} \end{array} \quad \Phi \rightleftharpoons \begin{pmatrix} \varphi_1; \\ \varphi_2; \\ \vdots \\ \varphi_{n_\Phi} \end{pmatrix},$$

respectively. Symbol λ marks the empty place. By $\varphi^{\triangledown\parallel^*}_{\triangleright}$ and Φ^* the system names are denoted. The other possible and precise symbolic representatives of the formula system are, for instance,

$$\varphi^{\triangledown\parallel}_{\triangleright} \rightleftharpoons \varphi^{\triangledown\parallel}_{\triangleright}\left\lfloor \varphi^{\triangledown}_{1\triangleright}, \varphi^{\triangledown}_{2\triangleright}, \ldots, \varphi^{\triangledown}_{n_{\varphi^{\triangledown\parallel}_{\triangleright}}\triangleright} \right\rfloor \textit{ and } \Phi \rightleftharpoons \Phi\lfloor \varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi} \rfloor, \textit{ and}$$

$$\varphi^{\triangledown\parallel}_{\triangleright} \rightleftharpoons \varphi^{\triangledown\parallel}_{\triangleright}\left\lceil \varphi^{\triangledown}_{1\triangleright}, \varphi^{\triangledown}_{2\triangleright}, \ldots, \varphi^{\triangledown}_{n_{\varphi^{\triangledown\parallel}_{\triangleright}}\triangleright} \right\rceil, \textit{ and } \Phi \rightleftharpoons \Phi\lceil \varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi} \rceil,$$

respectively. The first form is the formula-like expression, while the second form is the concerning-like expression of a formula system. □

The concerning-like expression of a formula system can be defined in an informationally complex way beyond the formula concept.

To the principle of formula system belongs the principle of common operands in system formulas. In fact, the reasonable definition of a system concerns common operands by which formulas are constituted as a system.

Sometimes it is also necessary that formula systems are closed under (common) operands, that each operand of the system is accessible from each other system operand (Železnikar, 1998). How can a closed formula system be defined?

Principle 6 *If in informational formulas* φ_1 *and* φ_2 *a common operand* α *appears, that is,* $\varphi_1\lfloor \ldots, \alpha, \ldots \rfloor$ *and* $\varphi_2\lfloor \ldots, \alpha, \ldots \rfloor$*, respectively, notation*

$$\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2 \quad \textit{or, simply,} \quad \varphi_1 \leftrightsquigarrow \varphi_2$$

will be used and read as formula φ_1 informs formula φ_2 *via* operand α *or, simply,* formula φ_1 is informationally linked to formula φ_2. *This operation is* informationally symmetric. *Thus,*

$$\left(\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2\right) \rightleftharpoons \left(\varphi_2 \overset{\alpha}{\leftrightsquigarrow} \varphi_1\right)$$

Transitivity of operator $\leftrightsquigarrow$ *can exist in the following way: if* α *is common to* φ_1 *and* φ_2*, and* β *is common to* φ_2 *and* φ_3*, that is,*

$$\varphi_1\lfloor \ldots, \alpha, \ldots \rfloor,\ \varphi_2\lfloor \ldots, \alpha, \ldots, \beta, \ldots \rfloor \ \textit{and}\ \ \varphi_3\lfloor \ldots, \beta, \ldots \rfloor,$$

then φ_1 *is linked informationally with* φ_3*. Formally,*

$$\left(\left(\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2\right) \wedge \left(\varphi_2 \overset{\beta}{\leftrightsquigarrow} \varphi_3\right)\right) \Longrightarrow \left(\varphi_1 \overset{\alpha,\beta}{\leftrightsquigarrow} \varphi_3\right)$$

Operator $\wedge$ *denotes informational conjunction (in fact, the operator of parallel informing, semicolon ';') and operator* $\Longrightarrow$ *informational implication.* □

Further, there can exist more than one common operand, e.g., α_i, α_j, α_k, $\ldots$, α_m in φ_1 and φ_2 in a transitive manner. In this case,

$$\left(\varphi_1 \overset{\alpha_i,\alpha_j,\alpha_k \ldots,\alpha_m}{\leftrightsquigarrow} \varphi_2\right) \rightleftharpoons \begin{pmatrix} \varphi_1 \overset{\alpha_i}{\leftrightsquigarrow} \varphi_{i'}; \\ \varphi_{i'} \overset{\alpha_j}{\leftrightsquigarrow} \varphi_{j'}; \\ \varphi_{j'} \overset{\alpha_k}{\leftrightsquigarrow} \varphi_{k'}; \\ \vdots \\ \varphi_{\ell'} \overset{\alpha_m}{\leftrightsquigarrow} \varphi_2 \end{pmatrix}$$

Transitivity of operator $\leftrightsquigarrow$ applies also to the case of more than one common operator through several formulas, and it can be defined from case to case.

Because common operands concern informing between formulas, the implication in Principle 6 can be expressed by means of a parallel (formula) system

$$\begin{pmatrix} \varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2; \\ \varphi_2 \overset{\beta}{\leftrightsquigarrow} \varphi_3 \end{pmatrix} \Longrightarrow (\varphi_1 \overset{\alpha,\beta}{\leftrightsquigarrow} \varphi_3)$$

Another significant feature follows from the last principle.

Consequence 2 *Let the linkages in a circular manner*

$$\varphi_1 \leftrightsquigarrow \varphi_2,\ \varphi_2 \leftrightsquigarrow \varphi_3,\ \ldots,\ \varphi_{m-1} \leftrightsquigarrow \varphi_m,\ \varphi_m \leftrightsquigarrow \varphi_1$$

be given. Then,

$$\varphi_1, \varphi_2, \ldots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m$$

This feature is called the reflexivity *of a circular informational linkage of formulas within (in the framework of) a formula system. Within a circular linkage, each operand is connected and accessible by each other operand, including itself.* □

Proof 2 We have to prove that

$$(\varphi_1, \varphi_2, \ldots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m) \Longrightarrow (\varphi_i \leftrightsquigarrow \varphi_j);\ i, j \in \{1, 2, \ldots, m\}$$

Within this conditionality also

$$\varphi_i \leftrightsquigarrow \varphi_i; \quad i \in \{1, 2, \ldots, m\}$$

holds in a transitive (consequently multiple-linkage) manner. Another evident meaning of the consequence is

$$\varphi_i \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m \text{ for all } i \in \{1, 2, \ldots, m\}$$

It means that φ_i is linked to each of $\varphi_1, \varphi_2, \ldots, \varphi_m$, including to itself (informational circularity). This proves the consequence. □

A formula system is closed if each operand of a system formula is linked to all formula operands. Natural language is a closed formula system. Closed formula systems are meaningful systems and can develop circularly by introduction of new operands.

7. Principles concerning informational schemes and graphs

The principle of informational scheme involves the concept of informational formula. A formula is a well-formed structure with operands and binary operators and, in this sense, being correctly parenthesized. The schematic presentation of a formula excludes the set parenthesis pairs and what remains is the so-called formula scheme. The procedure of deleting parenthesis pairs is called schematizing, denoted by $\mathfrak{S}$. Schematizing a formula means to have a rough overview of the sequence of operands and operators. Sentences in a natural language are often structures between a scheme and formula, especially in a spoken language, where the meaning of the sentence can be ambiguous or misunderstood.

To be more precise, schematizing means also the inverse procedure when constructing schemes out of an informational graph, moving along a graph path, loop, or complex structure. This sort of schematizing will be discussed in the case of solution concerning an operand which occurs in a graph. The graph is usually the result of schematizing a formula system, or several formula systems are fused (alloyed) into the single graph.

Principle 7 *If* $\varphi_\triangleright^\triangledown \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\varphi_\triangleright^\triangledown}} \right\rfloor$ *is a formula, its scheme is denoted by* $\mathfrak{S}\left\lceil \varphi_\triangleright^\triangledown \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\varphi_\triangleright^\triangledown}} \right\rfloor \right\rceil$ *or, shortly,* $\mathfrak{S}\left\lceil \varphi_\triangleright^\triangledown \right\rceil$. *The formula scheme results from the formula by omitting the parenthesis pairs.* □

What remains after the schematizing of a formula is a sequence of operands and operators. While the notation $\varphi_\triangleright^\triangledown \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\varphi_\triangleright^\triangledown}} \right\rfloor$ does not say anything about the concrete parenthesizing, its scheme is uniquely determined by $\triangleright \in \{\rightarrow, \leftarrow, \rightleftarrows, (\rightarrow, \leftarrow)\}$ and $\triangledown \in \{\lambda, \circlearrowleft\}$. The possible schemes are

$$\mathfrak{S}\lceil \varphi_\rightarrow \lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\rightarrow} \rfloor \rceil \rightleftharpoons (\alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_\rightarrow});$$
$$\mathfrak{S}\left\lceil \varphi_\rightarrow^\circlearrowleft \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\rightarrow^\circlearrowleft} \right\rfloor \right\rceil \rightleftharpoons \left(\alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_\rightarrow^\circlearrowleft} \models \alpha_1\right);$$
$$\mathfrak{S}\lceil \varphi_\leftarrow \lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\leftarrow} \rfloor \rceil \rightleftharpoons (\alpha_1 \Dashv \alpha_2 \Dashv \ldots \Dashv \alpha_{n_\leftarrow});$$
$$\mathfrak{S}\left\lceil \varphi_\leftarrow^\circlearrowleft \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\leftarrow^\circlearrowleft} \right\rfloor \right\rceil \rightleftharpoons \left(\alpha_1 \Dashv \alpha_2 \Dashv \ldots \Dashv \alpha_{n_\leftarrow^\circlearrowleft} \Dashv \alpha_1\right);$$
$$\mathfrak{S}\lceil \varphi_\rightleftarrows \lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\rightleftarrows} \rfloor \rceil \rightleftharpoons (\alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_\rightleftarrows} \models \alpha_{n_\rightleftarrows - 1} \models \ldots \models \alpha_2 \models \alpha_1);$$
$$\mathfrak{S}\left\lceil \varphi_\rightleftarrows^\circlearrowleft \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_\rightleftarrows^\circlearrowleft} \right\rfloor \right\rceil \rightleftharpoons \left(\alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_\rightleftarrows^\circlearrowleft} \models \alpha_1 \models \alpha_{n_\rightleftarrows^\circlearrowleft} \models \ldots \models \alpha_2 \models \alpha_1\right);$$
$$\mathfrak{S}\lceil \varphi_{\rightarrow,\leftarrow} \lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\rightarrow,\leftarrow}} \rfloor \rceil \rightleftharpoons \begin{pmatrix} \alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_{\rightarrow,\leftarrow}}; \\ \alpha_1 \Dashv \alpha_2 \Dashv \ldots \Dashv \alpha_{n_{\rightarrow,\leftarrow}} \end{pmatrix};$$
$$\mathfrak{S}\left\lceil \varphi_{\rightarrow,\leftarrow}^\circlearrowleft \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\rightarrow,\leftarrow}^\circlearrowleft} \right\rfloor \right\rceil \rightleftharpoons \begin{pmatrix} \alpha_1 \models \alpha_2 \models \ldots \models \alpha_{n_{\rightarrow,\leftarrow}^\circlearrowleft} \models \alpha_1; \\ \alpha_1 \Dashv \alpha_2 \Dashv \ldots \Dashv \alpha_{n_{\rightarrow,\leftarrow}^\circlearrowleft} \Dashv \alpha_1 \end{pmatrix}$$

The split biserial schemes (subscript $\rightarrow, \leftarrow$) are scheme systems with two subschemes. Circular schemes are longer than the adequate non-circular schemes. The length $\ell_\triangleright^\triangledown$ represents the number of operators in the scheme or formula. For each scheme, $\frac{1}{\ell_\triangleright^\triangledown + 1} \binom{2\ell_\triangleright^\triangledown}{\ell_\triangleright^\triangledown}$ different formulas can be parenthesized. Thus, for the split biserial scheme, $\left(\frac{1}{\ell_\triangleright^\triangledown + 1} \binom{2\ell_\triangleright^\triangledown}{\ell_\triangleright^\triangledown}\right)^2$ different split biserial systems are possible.

Principle 8 *If* $\varphi_{\triangleright}^{\triangledown\|} \rightleftharpoons \left(\varphi_{1\triangleright}^{\triangledown}, \varphi_{2\triangleright}^{\triangledown}, \ldots, \varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown}\right)$ *is a uniform formula system and* $\Phi \rightleftharpoons (\varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi})$ *a mixed formula system, the scheme is determined by*

$$\mathfrak{S}\left\lceil \varphi_{\triangleright}^{\triangledown\|} \right\rceil \rightleftharpoons \left(\mathfrak{S}\left\lceil \varphi_{1\triangleright}^{\triangledown} \right\rceil;\ \mathfrak{S}\left\lceil \varphi_{2\triangleright}^{\triangledown} \right\rceil;\ \ldots;\ \mathfrak{S}\left\lceil \varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown} \right\rceil\right) \quad \text{and}$$
$$\mathfrak{S}\lceil \Phi \rceil \rightleftharpoons (\mathfrak{S}\lceil \varphi_1 \rceil;\ \mathfrak{S}\lceil \varphi_2 \rceil;\ \ldots;\ \mathfrak{S}\lceil \varphi_{n_\Phi} \rceil),$$

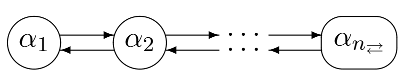
respectively. □

Graph is another, the most lose informational organization of operands and operators. In a graph, the original structure of formulas and formula systems is entirely lost. The aim of the graph is to enable the construction of formulas and formula systems in a new way, moving along the different paths of the graph and, then, separating different schemes into scheme system, rotating and parenthesizing schemes, and finally, identify the well-formed formulas and formula systems by parenthesizing.

Principle 9 *For a formula* φ*, considering its scheme* $\mathfrak{S}\lceil \varphi \rceil$*, the primitive formula system* φ' *can be constructed, consisting of all possible primitive transitions* $\alpha_i \models \alpha_{i+1}$ *obtained from the formula scheme, where* $i = 1, 2, \ldots, n_\varphi - 1$*. In case of a circular formula, additionally,* $\varphi_{n_\varphi} \models \alpha_1$*. For the formula graph* $\mathfrak{G}\lceil \varphi \rceil$*,* $\mathfrak{G}\lceil \varphi \rceil \rightleftharpoons \varphi'$*. A formula graph is equivalently represented by the primitive formula system.* □

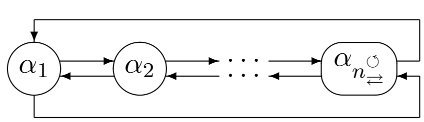
Evidently, for instance, the circular graphs for formulas $\overset{\circlearrowleft}{\varphi}_{\rightarrow}$, $\overset{\circlearrowleft}{\varphi}_{\leftarrow}$, $\overset{\circlearrowleft}{\varphi}_{\rightleftarrows}$, and $\overset{\circlearrowleft}{\varphi}_{\rightarrow,\leftarrow}$, can be presented in the 'graphical' form as shown in Figure 1. This

(A) $\mathfrak{G}\lceil \varphi_{\rightleftarrows} \lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n_{\rightleftarrows}} \rfloor \rceil$



$n_{\rightleftarrows} = n_{\rightarrow} = n_{\leftarrow} = n_{\rightarrow,\leftarrow}$;
$\mathfrak{G}\lceil \varphi_{\rightarrow} \rceil \prec \mathfrak{G}\lceil \varphi_{\rightleftarrows} \rceil; \mathfrak{G}\lceil \varphi_{\leftarrow} \rceil \prec \mathfrak{G}\lceil \varphi_{\rightleftarrows} \rceil$;
$\mathfrak{G}\lceil \varphi_{\rightarrow,\leftarrow} \rceil \equiv \mathfrak{G}\lceil \varphi_{\rightleftarrows} \rceil$

(B) $\mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightleftarrows} \left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{\overset{\circlearrowleft}{n}_{\rightleftarrows}} \right\rfloor \right\rceil$



$\overset{\circlearrowleft}{n}_{\rightleftarrows} = \overset{\circlearrowleft}{n}_{\rightarrow} = \overset{\circlearrowleft}{n}_{\leftarrow} = \overset{\circlearrowleft}{n}_{\rightarrow,\leftarrow}$;
$\mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightarrow} \right\rceil \prec \mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightleftarrows} \right\rceil; \mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\leftarrow} \right\rceil \prec \mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightleftarrows} \right\rceil$;
$\mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightarrow,\leftarrow} \right\rceil \equiv \mathfrak{G}\left\lceil \overset{\circlearrowleft}{\varphi}_{\rightleftarrows} \right\rceil$

Figure 1: Subgraphs and graphs for standard informational formulas.

figure demonstrates the concept of the subgraph. For the upper part of the graph $\mathfrak{G}\lceil\varphi_{\rightleftarrows}\rceil$ in (A), evidently, $\mathfrak{G}\lceil\varphi_{\rightarrow}\rceil, \mathfrak{G}\lceil\varphi_{\leftarrow}\rceil \prec \mathfrak{G}\lceil\varphi_{\rightleftarrows}\rceil$ and $\mathfrak{G}\left\lceil\varphi_{\rightarrow,\leftarrow}^{\circlearrowleft}\right\rceil \equiv \mathfrak{G}\left\lceil\varphi_{\rightleftarrows}^{\circlearrowleft}\right\rceil$, etc. Lastly, if $n_{\rightleftarrows} = n_{\rightleftarrows}^{\circlearrowleft}$, also $\mathfrak{G}\lceil\varphi_{\rightleftarrows}\rceil \prec \mathfrak{G}\left\lceil\varphi_{\rightleftarrows}^{\circlearrowleft}\right\rceil$.

Principle 10 *For a uniform formula system* $\varphi_{\triangleright}^{\triangledown\|} \rightleftharpoons \left(\varphi_{1\triangleright}^{\triangledown}; \varphi_{2\triangleright}^{\triangledown}; \ldots; \varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown}\right)$ *and a mixed formula system* $\Phi \rightleftharpoons (\varphi_1; \varphi_2; \ldots; \varphi_{n_\Phi})$, *considering formula schemes* $\mathfrak{S}\left\lceil\varphi_{1\triangleright}^{\triangledown}\right\rceil, \mathfrak{S}\left\lceil\varphi_{2\triangleright}^{\triangledown}\right\rceil, \ldots, \mathfrak{S}\left\lceil\varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown}\right\rceil$ *and* $\mathfrak{S}\lceil\varphi_1\rceil, \mathfrak{S}\lceil\varphi_2\rceil, \ldots, \mathfrak{S}\lceil\varphi_{n_\Phi}\rceil$, *the primitive formula systems* $\varphi_{1\triangleright}^{\triangledown\prime}, \varphi_{2\triangleright}^{\triangledown\prime}, \ldots, \varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown\prime}$ *and* $\varphi_1', \varphi_2', \ldots, \varphi_{n_\Phi}'$ *can be constructed, consisting of all possible primitive transitions* $\alpha_{ij} \models \alpha_{i,j+1}$, $i = 1, 2, \ldots, n_{\varphi_{\triangleright}^{\triangledown\|}}$, $j = 1, 2, \ldots, n_{\varphi_{i\triangleright}^{\triangledown}}$ *and* $i = 1, 2, \ldots, n_\Phi$, $j = 1, 2, \ldots, n_{\varphi_i}$, *obtained from the formula schemes, respectively. In case of a circular formula, additionally,* $\alpha_{n_{\varphi_{i\triangleright}^{\triangledown}}} \models \alpha_{i1}$ *and* $\alpha_{\varphi_i} \models \alpha_{i1}$. *For the formula system graph* $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\triangledown\|}\right\rceil \rightleftharpoons \varphi_{\triangleright}^{\triangledown\|\prime}$ *and* $\mathfrak{G}\lceil\Phi\rceil \rightleftharpoons \Phi'$, *where* $\varphi_{\triangleright}^{\triangledown\|\prime} \rightleftharpoons \left(\varphi_{1\triangleright}^{\triangledown\prime}; \varphi_{2\triangleright}^{\triangledown\prime}; \ldots; \varphi_{n_{\varphi_{\triangleright}^{\triangledown\|}}\triangleright}^{\triangledown\prime}\right)$ *and* $\Phi' \rightleftharpoons \left(\varphi_1'; \varphi_2'; \ldots; \varphi_{n_\Phi}'\right)$, *respectively.* □

In a system graph, the formula-parenthesized structures and system-semicolon structures alloy and become unrecognizable and irreversible. To use such a graph for informational purposes, the graph paths have to be schematized, and the obtained schemes parenthesized. In different ways, the graph can be covered by different schematizing, delivering different sorts of meaning by parenthesizing the concrete schemes.

8. Operand rotation and solution

A circular informational formula, $\varphi_{\triangleright}^{\circlearrowleft}$, exhibits the circular causality of its operands. This phenomenon becomes evident in the circular formula scheme $\mathfrak{S}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\right\rceil$ and its circular graph (loop), where the so-called feedback path leads from the end to the beginning of the serial graph. On the level of the circular formula scheme, the idea of the circular causal relativity becomes evident and the question of the reasonableness of operand rotation within the loop comes in the foreground. That is to say, by the operand rotation in a circular formula scheme, the circular causality of operands remains preserved.

It is certainly to stress that between a formula of length ℓ_φ and its scheme there is an essential difference. The formula scheme represents a general concept satisfying the schematizing of $\frac{1}{\ell_\varphi+1}\binom{2\ell_\varphi}{\ell_\varphi}$ formulas. A formula is usually an informational interpretation of the leftmost operand by the remaining operands, maybe a definition of the leftmost operand, for instance. Rotating an operand in the formula scheme (or its graph) means that a trial of expressing a non-leftmost operand by the remaining operands in the loop is made. In this way, through parenthesizing the result of an operand rotation in a circular scheme, a new formula is obtained, which expresses the new leftmost operand by the remaining operands. We can say, that the new formula is a solution for the new leftmost operand, expressing it by the remaining operands in a circular way.

Principle 11 *Let $\mathfrak{R}$ mark the general procedure of operand α_i rotation in a circular scheme $\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_1,\ldots,\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}\right\rfloor\right]$. Let operand α_i be rotated to the leftmost place of the scheme. The result is denoted by*

$$\mathfrak{R}\left[\alpha_i,\,\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_1,\ldots,\alpha_{i-1},\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}\right\rfloor\right]\right] \rightleftharpoons \mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}},\alpha_1,\ldots,\alpha_{i-1}\right\rfloor\right].$$

In this case, formula $\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}},\alpha_1,\ldots,\alpha_{i-1}\right\rfloor$ is one of the possible $\frac{1}{\ell_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}+1}\binom{2\ell_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}{\ell_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}$ formulas obtained from the new scheme by parenthesizing. This formula is called the solution for operand α_i concerning the original formula $\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_1,\ldots,\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}\right\rfloor$. □

The solution upon operand α_i concerning formula scheme can be denoted explicitly by $\mathfrak{s}_{\text{solution}}\left[\alpha_i,\,\overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_1,\ldots,\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}}\right\rfloor\right]$ or, in a direct form, as definition for α_i, that is, $\alpha_i \rightleftharpoons \overset{\circlearrowleft}{\varphi_{\triangleright}}\left\lfloor\alpha_i,\ldots,\alpha_{n_{\overset{\circlearrowleft}{\varphi_{\triangleright}}}},\alpha_1,\ldots,\alpha_{i-1}\right\rfloor$.

Principle 12 *Let the formula system schemes*

$$\mathfrak{S}\left[\overset{\circlearrowleft\|}{\varphi_{\triangleright}}\left\lfloor\overset{\circlearrowleft}{\varphi_{1\triangleright}},\overset{\circlearrowleft}{\varphi_{2\triangleright}},\ldots,\varphi_{n_{\overset{\circlearrowleft\|}{\varphi_{\triangleright}}}}\right\rfloor\right] \rightleftharpoons \left(\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{1\triangleright}}\right];\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{2\triangleright}}\right];\ldots;\mathfrak{S}\left[\varphi_{n_{\overset{\circlearrowleft\|}{\varphi_{\triangleright}}}}\right]\right)$$
$$\text{and}\quad \mathfrak{S}\left[\Phi\left\lfloor\overset{\circlearrowleft}{\varphi_1},\overset{\circlearrowleft}{\varphi_2},\ldots,\overset{\circlearrowleft}{\varphi_{n_\Phi}}\right\rfloor\right] \rightleftharpoons \left(\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_1}\right];\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_2}\right];\ldots;\mathfrak{S}\left[\overset{\circlearrowleft}{\varphi_{n_\Phi}}\right]\right)$$

be given. Let operand β occur in formulas $\overset{\circlearrowleft}{\varphi_{i\triangleright}}\lfloor\ldots,\beta,\ldots\rfloor$, $i=1,2,\ldots,n_{\overset{\circlearrowleft\|}{\varphi_{\triangleright}}}$ and $\overset{\circlearrowleft}{\varphi_j}\lfloor\ldots,\beta,\ldots\rfloor$, $j=1,2,\ldots,n_\Phi$. Let β be rotated to the leftmost place *of the system schemes. The result of schematic rotation is*

$$\mathfrak{R}\left\lceil\beta,\mathfrak{S}\left\lceil\varphi_{\triangleright}^{\circlearrowleft\|}\right\rceil\right\rceil \rightleftharpoons \begin{pmatrix}\mathfrak{R}\left\lceil\mathfrak{S}\left\lceil\varphi_{i\triangleright}^{\circlearrowleft}\lfloor\ldots,\beta,\ldots\rfloor\right\rceil\right\rceil \rightleftharpoons \mathfrak{S}\left\lceil\varphi_{i\triangleright}^{\circlearrowleft}\lfloor\beta,\ldots\rfloor\right\rceil; \\ i=1,2,\ldots,n^{\circlearrowleft}_{\varphi_{\triangleright}^{\circlearrowleft\|}}\end{pmatrix}$$

$$\textit{and}\ \ \mathfrak{R}\lceil\beta,\mathfrak{S}\lceil\Phi\rceil\rceil \rightleftharpoons \begin{pmatrix}\mathfrak{R}\left\lceil\mathfrak{S}\left\lceil\varphi_{j}^{\circlearrowleft}\lfloor\ldots,\beta,\ldots\rfloor\right\rceil\right\rceil \rightleftharpoons \mathfrak{S}\left\lceil\varphi_{j}^{\circlearrowleft}\lfloor\beta,\ldots\rfloor\right\rceil; \\ j=1,2,\ldots,n_{\Phi}\end{pmatrix}.$$

From schematized formula system $\mathfrak{R}\left\lceil\beta,\mathfrak{S}\left\lceil\varphi_{\triangleright}^{\circlearrowleft\|}\right\rceil\right\rceil$ *and* $\mathfrak{R}\lceil\beta,\mathfrak{S}\lceil\Phi\rceil\rceil$, *respectively,*

$$\prod_{i=1}^{n^{\circlearrowleft}_{\varphi_{\triangleright}^{\circlearrowleft\|}}} \frac{1}{n^{\circlearrowleft}_{\varphi^{\circlearrowleft}_{i\triangleright}}+1}\binom{2n^{\circlearrowleft}_{\varphi^{\circlearrowleft}_{i\triangleright}}}{n^{\circlearrowleft}_{\varphi^{\circlearrowleft}_{i\triangleright}}} \quad \textit{and} \quad \prod_{j=1}^{n_{\Phi}} \frac{1}{n^{\circlearrowleft}_{\varphi_j}+1}\binom{2n^{\circlearrowleft}_{\varphi_j}}{n^{\circlearrowleft}_{\varphi_j}}$$

different formula systems can be parenthesized. Each parenthesized system can represent the solution concerning operand α_i *and* α_j, *respectively.* □

In this sense, the solution concerning an operand (also formula, formula system) means the acquiring of meaning pertaining to the named operand, e.g., α, φ^*, $\varphi_{\triangleright}^{\circlearrowleft\|^*}$, and Φ^*.

9. The principle of informational decomposition

Why decomposition and how it is founded in the existing informational universe? Decomposition concerns the question what kind of meaning is hidden behind an informational entity. In the world of human life complex information exists. Here and there, informational lumps are around and can be organized to represent meaning of something. To acquire meaning pertaining to an emergent informational entity, the entity has to be interpreted, explained, enriched according to its intention, its initial intentional meaning, that is, decomposed. Decomposition organizes informational lumps into a reasonable meaning concerning an informational entity. Such entities emerge in conscious systems and develop informationally by various decompositions happening as an informational orchestration of events.

The word 'decomposition' means that in a complex system of emerging language the possibility exist to express entities of the language by the use of other entities and by the entities themselves (circular language causality, tautology, and the like). In this view, decomposition resolves and disintegrates intentionally the possible meaning, corresponding to its object within the language and puts the result of decomposition into the concrete informational structure (e.g., $\mathfrak{Z}$-language) of the entity under investigation. Decomposition is a metaphor for analysis and generation concerning an object in the existing informational environment.

At the first glance, decomposition of informational entity seems to be a kind of deus ex machina which happens to the entity being decomposed. Namely, decomposition means an informational emergence (e.g., of a new operand name), extension, change and/or reduction of the entity. To perform its creative function, decomposition must have on disposal its own creative conscious ability of analysis and generation, and the complexity (overview of possibilities) within an informational domain.

Decomposition is an emergent, complex, and consciously organized informational entity (formula system) for decomposition of operands, formulas, and formula systems. It analyzes informational situations and generates extensions, changes and reductions upon the entity under investigation, its possible interpretation. Decomposition chooses, according to the object and its informational situation in a context, the corresponding intention for the flow of decomposition, decides spontaneously upon it. It investigates possibilities in the framework of the surrounding conscious system with the aim (intention) to develop meaningly the entity under decomposition. In this function, decomposition analyses, compares, decides and generates a concrete form of entity decomposition. Informational decomposition belongs to the so-called *informonic* informational entities (see Section 11).

Decomposition by itself is a metainformational entity for organizing the meaning according to an entity's intention. Intention roots in the possibility and need to stretch the initial meaning of a pure name (or a complex informational entity), which has to be put into an informational texture, that is, explained by the emerging structure in which other informational entities occur. The general principle of informational decomposition can be formulated in the following way.

Principle 13 *Informational decomposition is an informational entity for acquiring the meaning pertaining to a named informational entity. In this view, decomposition complexly develops operands, formulas, and formula systems explaining the entity by other entities and itself (circular decomposition). Recursively, decomposition of decomposition is informationally regular.* □

Particular general decompositions $\Delta_{\triangleright}^{\triangledown}$ and $\Delta_{\triangleright}^{\triangledown\|}$ concern type ($\triangledown \in \{\lambda, \circlearrowleft\}$, $\triangleright \in \{\rightarrow, \leftarrow, \rightleftarrows, (\rightarrow, \leftarrow)\}$) and object (simple operand α, formula $\varphi_{\triangleright}^{\triangledown}$, formula system $\varphi_{\triangleright}^{\triangledown\|}$ and Φ). In this sense, 64 general decompositions can be distinguished, 16 for each of operand, formula, uniform formula system, and mixed formula system.

Let's describe colloquially and formally the subject and difference between the decomposition of an operand, a formula and formula system. Decomposing a simple operand (name) means to form a formula proceeding from the operand itself. If decomposition is a parallel one, several formulas with the same initial operand will be generated and fused into formula system. Decomposing a formula, it has to be subdivided into its subformulas, that is, in everything enclosed into the parenthesis pairs plus the simple operands and the formula itself. The number of subformulas ψ_i in a formula $\varphi^{\triangledown\varphi}_{\triangleright_\varphi}$, $\psi_i \Subset \varphi^{\triangledown\varphi}_{\triangleright_\varphi}$, is determined by

$$\mathbb{L}^{\triangledown\varphi}_{\triangleright_\varphi} = \begin{cases} \ell^{\triangledown\varphi}_{\triangleright_\varphi} + n^{\triangledown\varphi}_{\triangleright_\varphi} & \text{for } \triangleright \in \{\rightarrow, \leftarrow, \rightleftarrows\}, \\ 2\ell^{\triangledown\varphi}_{\rightarrow,\leftarrow} + n^{\triangledown\varphi}_{\rightarrow,\leftarrow}, & \end{cases}$$

where the number of proper subformulas corresponds to $\ell^{\triangledown\varphi}_{\triangleright_\varphi}$ and $n^{\triangledown\varphi}_{\triangleright_\varphi}$ is the number of operands according to notation $\varphi^{\triangledown\varphi}_{\triangleright_\varphi}\left\lceil \alpha_1, \alpha_2, \ldots, \alpha_{n^{\triangledown\varphi}_{\triangleright_\varphi}} \right\rceil$. In case of $\varphi^{\triangledown\varphi}_{\rightarrow,\leftarrow}$ there are two equally long formulas (the serial and reverse serial) with the same operands.

Principle 14 *General decomposition of a formula* $\varphi^{\triangledown\varphi}_{\triangleright_\varphi}\left\lfloor \alpha_1, \alpha_2, \ldots, \alpha_{n^{\triangledown\varphi}_{\triangleright_\varphi}} \right\rfloor$ *is completely determined by*

$$\Delta^{\triangledown}_{\triangleright}\left\lceil \varphi^{\triangledown\varphi}_{\triangleright_\varphi} \right\rceil \rightleftharpoons \left(\Delta^{\triangledown}_{\triangleright i}\lceil \psi_i \rceil;\ \psi_i \Subset \varphi^{\triangledown\varphi}_{\triangleright_\varphi};\ i = 1, 2, \ldots, \mathbb{L}^{\triangledown\varphi}_{\triangleright_\varphi} \right).$$

For a uniform formula system $\varphi^{\triangledown\varphi\|}_{\triangleright_\varphi} \rightleftharpoons \left(\varphi^{\triangledown\varphi}_{\triangleright_\varphi 1}, \varphi^{\triangledown\varphi}_{\triangleright_\varphi 2}, \ldots, \varphi^{\triangledown\varphi}_{\triangleright_\varphi n^{\triangledown\varphi\|}_{\triangleright_\varphi}} \right)$, *the general decomposition is*

$$\Delta^{\triangledown}_{\triangleright}\left\lceil \varphi^{\triangledown\varphi\|}_{\triangleright_\varphi} \right\rceil \rightleftharpoons \begin{pmatrix} \Delta^{\triangledown}_{\triangleright}\left\lceil \varphi^{\triangledown\varphi\|^*}_{\triangleright_\varphi} \right\rceil;\ \ \Delta^{\triangledown}_{\triangleright ij_i}\lceil \psi_{ij_i} \rceil;\ \ \psi_{ij_i} \Subset \varphi^{\triangledown\varphi}_{\triangleright_\varphi i}; \\ i = 1, 2, \ldots, n^{\triangledown\varphi\|}_{\triangleright_\varphi};\ j_i = 1, 2, \ldots, \mathbb{L}^{\triangledown\varphi}_{\triangleright_\varphi i} \end{pmatrix}.$$

Finally, for a parallel general decomposition of a mixed formula system $\Phi \rightleftharpoons (\varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi})$*, the decomposed system is, formally,*

$$\Delta^{\triangledown\|}_{\triangleright}\lceil \Phi \rceil \rightleftharpoons \begin{pmatrix} \Delta^{\triangledown\|}_{\triangleright}\lceil \Phi^* \rceil;\ \ \Delta^{\triangledown\|}_{\triangleright ij_i}\lceil \psi_{ij_i} \rceil;\ \ \psi_{ij_i} \Subset \varphi_i; \\ i = 1, 2, \ldots, n_\Phi;\ j_i = 1, 2, \ldots, \mathbb{L}_{\varphi_i} \end{pmatrix}.$$

Formula systems are usually named, so their names $\varphi^{\triangledown\varphi\|^*}_{\triangleright_\varphi}$ *and* Φ^* *have to be specifically decomposed too. Other cases of decomposition fall into the domain of the presented three sorts of general decomposition.* □

Inductively, a parallel general decomposition (superscript $\|$) of a mixed formula system $\Phi \rightleftharpoons (\varphi_1, \varphi_2, \ldots, \varphi_{n_\Phi})$ would deliver a complex system of the form

$$\Delta_{\triangleright}^{\triangledown\|}\lceil\Phi\rceil \rightleftharpoons \begin{pmatrix} \Delta_{\triangleright i_0}^{\triangledown}\lceil\Phi^*\rceil; \;\; i_0 = 1, 2, \ldots, n_{\Delta_{\triangleright i_0}^{\triangledown\|}\lceil\Phi^*\rceil}; \\ \Delta_{\triangleright ij_i k_{ij_i}}^{\triangledown}\lceil\psi_{ij_i}\rceil; \;\; \psi_{ij_i} \Subset \varphi_i; \; i = 1, 2, \ldots, n_\Phi; \\ \quad j_i = 1, 2, \ldots, \mathbb{L}_{\varphi_i}; \; k_{ij_i} = 1, 2, \ldots, n_{\Delta_{\triangleright ij_i k_{ij_i}}^{\triangledown}\lceil\psi_{ij_i}\rceil} \end{pmatrix}.$$

Evidently, the recipe for the decomposition of mixed formula system considers the following steps:

(1) $n_{\Delta_{\triangleright i_0}^{\triangledown\|}\lceil\Phi^*\rceil}$ different decompositions of system name Φ^*,
(2) n_Φ system formulas φ_i,
(3) $\mathbb{L}_{\varphi_i}$ subformulas ψ_{ij_i} in each formula φ_i of system Φ and, finally,
(4) $n_{\Delta_{\triangleright ij_i k_{ij_i}}^{\triangledown}\lceil\psi_{ij_i}\rceil}$ different decompositions of subformulas ψ_{ij_i} of system formulas φ_i of system Φ (in fact, parallel decomposition).

Other definitions of parallel decomposition of systems are possible. For instance, to make the parallel decomposition of system formulas φ_i first or, even, to make parallel decomposition of both system formulas and their subformulas (double parallel decomposition). Informational formalism offers rigorous concepts of different sorts of parallel decomposition.

Other sorts of decompositions can be introduced, for instance, metaphysicalistic, communicational, and informonic decomposition. They figure essentially as informational shells and graph envelopes limiting informational entities in their domains and possibilities of meaning (see the next section). However, decompositions are general informational entities overlooking the interior meaning (structure, organization) of an entity, the exterior meaning in other entities, comparing the both, and deciding on the course of an instantaneous decomposition concerning the intention of the decomposed entity and its operands. In this view, the decomposing entity is a conscious component performing as a local or global informon.

10. Informational shell and graph envelope

An informational shell is determined to be any circular formula or circular formula system delivering directly or transitively (through more than one formula) a graph envelope, the outmost loop of the shell graph. Proceeding from the graph, its envelope can be expressed explicitly by a circular formula scheme, that is, can be schematized circularly. For instance, a scientific discipline is determined by specific principles (axioms, rules of derivation) forming together with the meaning of a natural language the shell of the discipline. The shell with envelope does not limit a developing domain of research to a self-sufficient and non-emergent meaning. New meaning can develop from the informational environment beyond the envelope via the common operands of the interior and exterior domain pertaining to an informational entity. Within the shell of an informational entity the meaning of the entity develops dynamically (emerges) and the envelope develops together with the shell.

Principle 15 *Informational shell is a circular decomposition of an (initial) operand setting its initial organization from which other decompositions can follow, extending the meaning of the operand. The initial decomposition of the named operand and, then, further decompositions of shell operands, extend to the interior of the shell enriching its structure, that is, the formula system representing the initial operand after decomposition steps.* □

Both non-parallel and parallel circular decomposition, that is, $\Delta_{\triangleright}^{\circlearrowleft}\lceil\xi\rceil$ and $\Delta_{\triangleright}^{\circlearrowleft\|}\lceil\xi\rceil$, respectively, depend on the sort of operand $\xi \in \left\{\alpha, \varphi_{\triangleright}^{\triangledown}, \varphi_{\triangleright}^{\triangledown\|}, \Phi\right\}$. For a simple operand α decomposition we have

$$\Delta_{\triangleright}^{\circlearrowleft}\lceil\alpha\rceil \rightleftharpoons \varphi_{\triangleright}^{\circlearrowleft}\left\lfloor\alpha, \alpha_1, \alpha_2, \ldots, \alpha_{n_{\triangleright}^{\circlearrowleft}}\right\rfloor \text{ and}$$
$$\Delta_{\triangleright}^{\circlearrowleft\|}\lceil\alpha\rceil \rightleftharpoons \left(\Delta_{\triangleright i}^{\circlearrowleft}\lceil\alpha\rceil;\ i = 1, 2, \ldots, n_{\Delta_{\triangleright}^{\circlearrowleft\|}\lceil\alpha\rceil}\right).$$

In the parallel case, $n_{\Delta_{\triangleright}^{\circlearrowleft\|}\lceil\alpha\rceil}$ different decompositions $\Delta_{\triangleright i}^{\circlearrowleft}\lceil\alpha\rceil$ of operand α are generated, representing $n_{\Delta_{\triangleright}^{\circlearrowleft\|}\lceil\alpha\rceil}$ formulas of meaning (interpretation, explanation) concerning the operand. These formulas enter into the shell of operand α building up together with existing formulas the operand α system of meaning.

For a formula $\varphi_{\triangleright_\varphi}^{\triangledown}$, the general non-parallel and parallel circular decomposition (superscripts $\circlearrowleft$ and $\circlearrowleft\|$, respecrively) is

$$\Delta_{\triangleright}^{\circlearrowleft}\left\lceil\varphi_{\triangleright_\varphi}^{\triangledown}\right\rceil \rightleftharpoons \left(\Delta_{\triangleright i}^{\circlearrowleft}\lceil\psi_i\rceil;\ \psi_i \Subset \varphi_{\triangleright_\varphi}^{\triangledown};\ i = 1, 2, \ldots, \mathbb{L}_{\varphi_{\triangleright_\varphi}^{\triangledown}}\right) \text{ and}$$
$$\Delta_{\triangleright}^{\circlearrowleft\|}\left\lceil\varphi_{\triangleright_\varphi}^{\triangledown}\right\rceil \rightleftharpoons \begin{pmatrix} \Delta_{\triangleright i j_i}^{\circlearrowleft}\lceil\psi_i\rceil;\ \psi_i \Subset \varphi_{\triangleright_\varphi}^{\triangledown};\ i = 1, 2, \ldots, \mathbb{L}_{\varphi_{\triangleright_\varphi}^{\triangledown}}; \\ j_i = 1, 2, \ldots, n_{\Delta_{\triangleright i j_i}^{\circlearrowleft}\lceil\psi_i\rceil} \end{pmatrix}.$$

A formula decomposition always considers the subformulas of the formula, including the simple formula operands and the formula itself (relation $\varphi_{\triangleright_\varphi}^{\triangledown} \Subset \varphi_{\triangleright_\varphi}^{\triangledown}$ is true). In case of parallel decomposition, first, all subformulas ψ_i in formula $\varphi^{\nabla}_{\triangleright_\varphi}$ are identified and, then, each subformula ψ_i is decomposed in a parallel way, that is, $\Delta^{\circlearrowleft}_{\triangleright ij_i}\lceil\psi_i\rceil$, where $j_i = 1, 2, \ldots, n_{\Delta^{\circlearrowleft}_{\triangleright ij_i}\lceil\psi_i\rceil}$ is the parameter of parallel decomposition for subformula ψ_i. Thus, the number of all decompositions amounts to $\sum_{i=1}^{\mathbb{L}_{\varphi^{\nabla}_{\triangleright_\varphi}}} n_{\Delta^{\circlearrowleft}_{\triangleright ij_i}\lceil\psi_i\rceil}$. Certainly, still more could be said in describing verbally the power of these two decomposition formalisms (see Section 18 in (Železnikar, 2002c)).

Metaphysicalistic decomposition $\mathfrak{M}^{\circlearrowleft\|}_{\triangleright}$ introduces the concepts of an informational entity α to possess the property of intentional informing $(\mathfrak{I}, \mathfrak{i})$, counter-informing $(\mathfrak{C}, \mathfrak{c})$, and informational embedding $(\mathfrak{E}, \mathfrak{e})$. Informational entity should inform (produce), counter-inform (counter-produce, oppose), and embed (legalize, stabilize) the produced and counter-produced (intentionally opposing) information into the body (shell, interior) of the entity. Metaphysicalistic shell concerning an operand (name with a basic meaning, the initially recognized intention) is defined by the six formula system

$$\mathfrak{M}^{\circlearrowleft\|}_{\triangleright}\lceil\alpha\rceil \rightleftharpoons \begin{pmatrix} \varphi^{\circlearrowleft}_{\triangleright}\lfloor\alpha, \mathfrak{I}_\alpha, \mathfrak{i}_\alpha, \mathfrak{C}_\alpha, \mathfrak{c}_\alpha, \mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor; \\ \varphi^{\circlearrowleft}_{\triangleright}\lfloor\mathfrak{I}_\alpha, \mathfrak{i}_\alpha, \mathfrak{C}_\alpha, \mathfrak{c}_\alpha\rfloor;\ \ \varphi^{\circlearrowleft}_{\triangleright}\lfloor\mathfrak{C}_\alpha, \mathfrak{c}_\alpha, \mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor; \\ \varphi^{\circlearrowleft}_{\triangleright}\lfloor\mathfrak{I}_\alpha, \mathfrak{i}_\alpha\rfloor;\ \ \varphi^{\circlearrowleft}_{\triangleright}\lfloor\mathfrak{C}_\alpha, \mathfrak{c}_\alpha\rfloor;\ \ \varphi^{\circlearrowleft}_{\triangleright}\lfloor\mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor \end{pmatrix}.$$

The graph for serial decomposition $\mathfrak{M}^{\circlearrowleft\|}_{\rightarrow}\lceil\alpha\rceil$, that is, $\mathfrak{G}\left\lceil\mathfrak{M}^{\circlearrowleft\|}_{\rightarrow}\lceil\alpha\rceil\right\rceil$, is presented in Figure 2. What is the clue of formula system $\mathfrak{M}^{\circlearrowleft\|}_{\rightarrow}\lceil\alpha\rceil$ recognizable

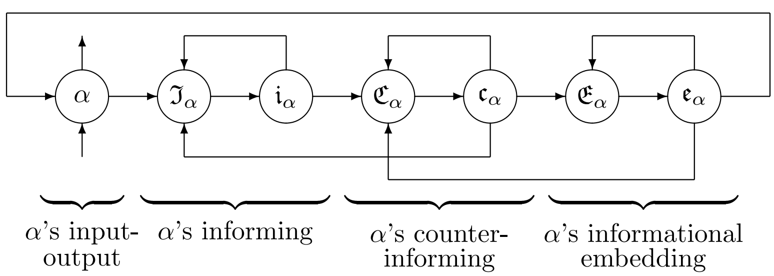


Figure 2: Graph of the metaphysicalistic standardized serial decomposition $\mathfrak{M}^{\circlearrowleft\|}_{\rightarrow}\lceil\alpha\rceil$ representing the formula system of six circular formulas (six loops).

transparently from the graph $\mathfrak{G}\left\lceil\mathfrak{M}^{\circlearrowleft\|}_{\rightarrow}\lceil\alpha\rceil\right\rceil$? Which kind of the background concept is lurking behind the metaphysicalistic decomposition?

In general, the task of a decomposition concerning an entity is to produce (generate), together with the entity itself (including it), a formula system representing an extended meaning to the entity being decomposed. At the beginning, in case of an initial shell, the basic mechanism is generated, being capable for further production of meaning concerning the initial intention of the entity. Included in its function, the instantaneous decomposition possesses the insight concerning the meaning, pertaining to the internal intention and its meaning developed up to now, and to the related meaning in the external entities. By $\mathfrak{M}_{\rightarrow}^{\circlearrowleft\|}\lceil\alpha\rceil$ it is said what the decomposition produces, however, the details how it comes to a concrete production of the formula system is not explained yet.

The structure presented in Figure 2 has its own reasonable meaning. The longest loop, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\alpha, \mathfrak{I}_\alpha, \mathfrak{i}_\alpha, \mathfrak{C}_\alpha, \mathfrak{c}_\alpha, \mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor\right\rceil$, includes all the operands of the initial decomposition. This loop becomes the envelope of the complex entity α, for which the meaning will be developed by further decompositions of the informing components. Additionally, some authenticity of informing to the operands and their inner circular linkages is granted. Thus, the three basic subsystems for informing, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\mathfrak{I}_\alpha, \mathfrak{i}_\alpha\rfloor\right\rceil$, conter-informing, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\mathfrak{C}_\alpha, \mathfrak{c}_\alpha\rfloor\right\rceil$, and informational embedding, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor\right\rceil$, inform authentically (circularly, spontaneously) in themselves, becoming responsible for the production of entities $\mathfrak{i}_\alpha$ (information, intention), $\mathfrak{c}_\alpha$ (counter-information, counter-intention), and $\mathfrak{e}_\alpha$ (embedding, determination) in further steps of the entity decomposition. To these loops, two middle-long loops are provided, $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\mathfrak{I}_\alpha, \mathfrak{i}_\alpha, \mathfrak{C}_\alpha, \mathfrak{c}_\alpha\rfloor\right\rceil$ and $\mathfrak{G}\left\lceil\varphi_{\triangleright}^{\circlearrowleft}\lfloor\mathfrak{C}_\alpha, \mathfrak{c}_\alpha, \mathfrak{E}_\alpha, \mathfrak{e}_\alpha\rfloor\right\rceil$, adjusting the discrepances and balances between informing $\mathfrak{I}_\alpha$ and counter-information $\mathfrak{c}_\alpha$ and between counter-informing $\mathfrak{C}_\alpha$ and information of embedding $\mathfrak{e}_\alpha$, respectively. By the middle-long loops, the authenticity between informing and counter-informing and between counter-informing and informational embedding is meant.

Definition of the graph envelope can now be formulated by the next principle.

Principle 16 *Graph envelope is a loop, which limits a subgraph representing an entity, in the following way: irrespective of the complexity of the entity, everything representing the meaning of the entity as such (locally) is linked within the envelope. This means that links (operand connections) to operands in the entity's environment lead solely via the envelope operands.* □

Informonic decomposition is a generalization of metaphysicalistic decomposition discussed in the next section.

11. Informon — a conscious informational entity

Informon is a universalization in complexity and conscious performance of informational entity informing in the informational cosmos. Operands informing within an entity's envelope (shell) are local informational entities being transformed and meaningly developed by decompositions of objects (operands) informing in the informational local and global environment. Each individual consciousness can be understood as an informational universe per se informing in informational cosmos together with other conscious and unconscious entities. However, the complexity and conscious organization of individual consciousness suffice for an entity within the conscious system to inform consciously. In this respect local and global informons can be distinguished.

Principle 17 *Informon is a complex and perplexed local and global formula system* $\underline{\boldsymbol{\alpha}}$ *and* $\widehat{\underline{\boldsymbol{\alpha}}}$*, respectively, named* α*, possessing a conscious structure of informational organization.* □

Informational complexity was not defined so far. It concerns perplexedness of system formulas through common operands, the number of system formulas, that is, system components, and the system functionality, for instance, a conscious organization. The structure pertains to the formula system expression, the organization of the perplexedness of meaning of operands and operators occurring in the system.

A general model of informonic decomposition is graphically presented

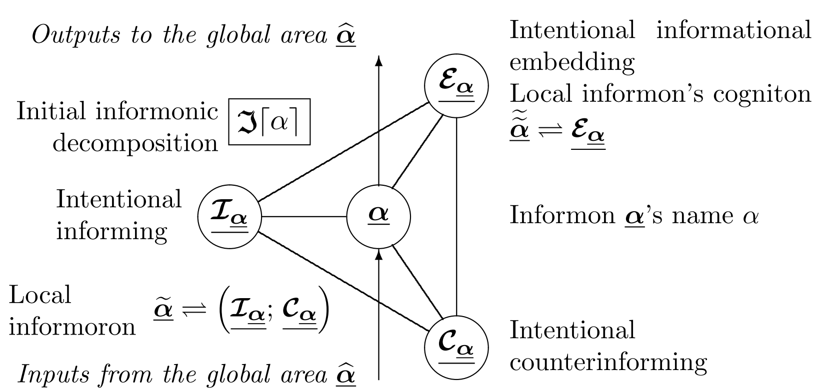


Figure 3: The possible graph structure presents a maximal biserial form, called also the bicircular informational supervenience of informing, counterinforming, and informational embedding (Železnikar, 2002c).

in Figure 3. Biserial means that paths between operands are bidirectional (operators $\models$ and $\Dashv$). Still the two new terms are introduced: *informoron* is a fusion of informonic informing and counter-informing while *cogniton* is informon-like embedding of acquired or emerged informational contents (formulas, formula systems). From the consciousness perspective, informoron is a fusion of intentional and emotional components, while cogniton is a constitution of cognitive components emerged during the informon's informing.

To the most important sort of informons belong informational decompositions. A decomposition is capable to capture the meaning (name, intention) of its operand irrespective of the operand's complexity. As a local and global informon it searches for the meaning pertaining to its operand in the interior and exterior of the entity represented by the operand. Evidently, in this respect, decomposition performs as an entity's constructing (teaching) agent, producing new meaning and instructing the operand about its meaning. Decomposition complexifies (enlarges, purifies, perplexes) the meaning of its operand. If in its initial name state the operand was informonically decomposed, further and numerous decompositions can follow and contribute to the turn where the operand becomes conscious, that is, becomes the informon.

The question arises what is formally reasonable and possible and how it can be explained when studying, for instance, general and metaphysicalistic decompositions. Certainly, the possible simple, local and global informonic general decompositions are $\Delta_{\triangleright}^{\triangledown}\lceil \underline{\boldsymbol{\alpha}} \rceil$, $\Delta_{\triangleright}^{\triangledown\|}\lceil \underline{\boldsymbol{\alpha}} \rceil$, $\underline{\boldsymbol{\Delta}_{\triangleright}^{\triangledown}\lceil \boldsymbol{\alpha} \rceil}$, $\underline{\boldsymbol{\Delta}_{\triangleright}^{\triangledown\|}\lceil \boldsymbol{\alpha} \rceil}$, $\underline{\boldsymbol{\Delta}_{\triangleright}^{\triangledown}\lceil \underline{\boldsymbol{\alpha}} \rceil}$, $\underline{\boldsymbol{\Delta}_{\triangleright}^{\triangledown\|}\lceil \underline{\boldsymbol{\alpha}} \rceil}$, $\Delta_{\triangleright}^{\triangledown}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil$, $\Delta_{\triangleright}^{\triangledown\|}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil$, $\underline{\widehat{\boldsymbol{\Delta}_{\triangleright}^{\triangledown}\lceil \boldsymbol{\alpha} \rceil}}$, $\underline{\widehat{\boldsymbol{\Delta}_{\triangleright}^{\triangledown\|}\lceil \boldsymbol{\alpha} \rceil}}$, $\underline{\widehat{\boldsymbol{\Delta}_{\triangleright}^{\triangledown}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil}}$, $\underline{\widehat{\boldsymbol{\Delta}_{\triangleright}^{\triangledown\|}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil}}$, generating the necessary and possible complexity of meaning. Possible metaphysicalistic decompositions concerning non-informonic and informonic entities can be systematically symbolized by $\underline{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \boldsymbol{\alpha} \rceil}$, $\underline{\widehat{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \boldsymbol{\alpha} \rceil}}$, $\underline{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \underline{\boldsymbol{\alpha}} \rceil}$, $\underline{\widehat{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \underline{\boldsymbol{\alpha}} \rceil}}$, $\underline{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil}$, $\underline{\widehat{\mathfrak{M}_{\triangleright}^{\circlearrowleft\|}\lceil \underline{\widehat{\boldsymbol{\alpha}}} \rceil}}$, etc.

12. Toward the philosophy of the informational and consciousness

Philosophy has long not recognized and ignored the significance of informational phenomena as a philosophical study, for instance, as a sort of phenomenology of informational entities. The question of the Being as informing was not raised explicitly within phenomenology, in the sense that that which is being informs, and as informing comes to the Being of consciousness. Informational externalism (Principle 1) and informational internalism (Principle 2) is something that seems to correspond to externalism and internalism of Being, expressed in the symbolic form as $\alpha \models\!\rightrightarrows [\,]$ and $[\,] \models\!\rightrightarrows \alpha$, respectively, where general operator or informational joker $\models$ (informs, is being informed) is replaced by the operator of Being or existence $\models\!\rightrightarrows$ (is, is being). These formulas of Being read α is (something) and (there) is α.

Operator $\models\!\rightrightarrows$ is a particular form of operator $\models$, namely $\models_{\text{be}}$ (the verb 'to be').

Another important philosophical view is the question of informational supervenience. In traditional philosophy, the correspondence concerning the physical, the informational, and the phenomenal is a problem of logical and natural supervenience (Chalmers, 1996; Železnikar, 1997). Informational supervenience concerns the tripartite domain of the informational and represents an informationalist view of the phenomenal or the conscious constituted by the informational, the counter-informational, and the informationally embedding. This concept of supervenience seems to be a reflection of human conscious experience recognizing the intentional nature of informing of entities in the domain of consciousness and particularly in human language, imagination (seeing the world) and other sensory experience. The general interplay of components in the tripartite informational domain, in the form of informational supervenience, is presented by a bicircular graph in Figure 4. The advantage of such a graphical scheme is that all of the

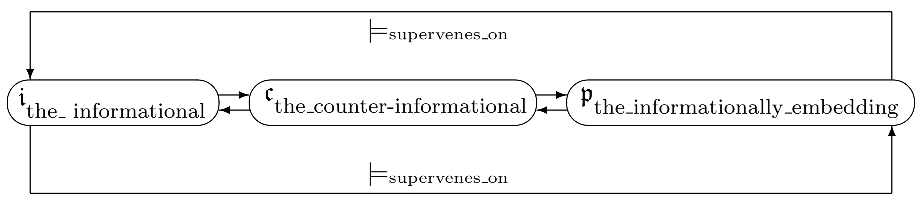


Figure 4: Graph of the bicircularly structured supervenience concerning the informational, the counter-informational, and the informationally embedding.

three components can directly impact each other. Evidently, the primitive formula system of the graph is

$$\Phi^{\circlearrowleft\prime}_{\rightleftarrows\text{informational\_supervenience}} \rightleftharpoons \left(\begin{array}{l} \mathfrak{i}_{\text{the\_counter-informational}} \models_{\text{supervenes\_on}} \mathfrak{i}_{\text{the\_informational}}; \\ \mathfrak{i}_{\text{the\_counter-informational}} \,=\!\mid_{\text{supervenes\_on}} \mathfrak{i}_{\text{the\_informational}}; \\ \mathfrak{p}_{\text{the\_informationally\_embedding}} \models_{\text{supervenes\_on}} \mathfrak{i}_{\text{the\_counter-informational}}; \\ \mathfrak{p}_{\text{the\_informationally\_embedding}} \,=\!\mid_{\text{supervenes\_on}} \mathfrak{i}_{\text{the\_counter-informational}}; \\ \mathfrak{i}_{\text{the\_informational}} \models_{\text{supervenes\_on}} \mathfrak{p}_{\text{the\_informationally\_embedding}}; \\ \mathfrak{i}_{\text{the\_informational}} \,=\!\mid_{\text{supervenes\_on}} \mathfrak{p}_{\text{the\_informationally\_embedding}} \end{array}\right)$$

The first impulse is given by the informational, representing a name with its meaning as a kind of intention at the very beginning and afterward when the decomposed informational entity becomes more and more meaningly developed. In the intention there is a conter-intentional impulse causing the counter-informing to that already informed. At last, intentionally and counter-intentionally generated information has to be recognized and in some way informationally embedded within the entity. In this way, the entity is being currently constituted, where the intentional part of the entity has the overview of the entity in whole, that is being informed by both parts of the supervenient system, so further cycles of informing can follow.

This model can be projected onto both everyday conscious experience and on the state-of-the-art in research of cognition (Dalgleish, Power, 1999) and emotions (Lewis, Haviland-Jones, 2000; Železnikar, 2002c). The informational part is understood to represent the intention being hidden in the meaning of the entity. The meaning will develop through decompositions of the entity during its circular informing through intentional counter-informing and intentional informational embedding. It is meant that the conter-informational part of the entity is represented by the emotional system of consciousness while the informationally embedding part represents the cognitive system. All of the three parts are linked together in a perplexed way, so they influence and control each other essentially up to the finest details. When reaching the sufficient complexity in the number of different operands, operators, their perplexity, and informational connectedness with the consciously complex inner and outer environment, the entity crosses the threshold dividing its unconscious and conscious informing. It reaches the state of self-consciousness or consciousness in general, so it can become its own informational developer with the ability to decompose itself consciously. This constitutes the most important hypothesis in the domain of consciousness and particularly artificial consciousness by which individual human consciousness will be understood, modelled and, in some respect, also functionally surpassed.

13. Conclusion

The aim of development of the philosophy and formalization of the informational was in the possibility to use informational principles and the innovative formalism at the design of consciousness models. Since 1987, the basic philosophical ground was searched and established (Železnikar, 1987) from which the initial formalization of informational concepts became possible. A special symbolism of formalization was systematically developed and became later possible by creation of standardized symbols within the AMS fonts in LaTeX. Without the new symbolism the endeavor of informational formalization and the verbal consequences originating in the formalism would not be possible.

In this sense, it was also possible to formalize the research concepts in the domain of cognition and emotions and gradually construct the basis (Železnikar, 2002c) from which more serious projects in complexity of conscious systems can start, taking into account the goal to design and implement artificially conscious systems for various applications. The paper remains on the way to introduce the newcomer into the innovative province of the informational by the most fundamental principles of externalism, internalism, and metaphysicalism. Another essential concept of the informational is decomposition as a complexly designed tool for development of meaning, by which the emergent nature of entities can be supported.

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